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Earthwork and Retaining Walls

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EARTHWORK
DESIGN OF RETAINING WALLS

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EARTHWORK

Serial 3072-2 Edition 1

FIELD MEASUREMENTS

INTRODUCTION

DEFINITIONS

1. Necessity of Cutting and Filling.—The economical operation of a railroad or of a highway requires that it be built within certain limits of grade. In the construction of railroads and highways, the irregularities of the surface of the earth must first be removed by cutting through high spots and filling over low spots. This excavation and filling, known as grading, or earthwork, is particularly heavy on railroads, which usually require easier grades and less abrupt changes of grade than can be allowed for highways.

2. Cut.—The earth or rock that is removed to permit placing the road below the natural surface of the ground is called cut, or excavation. The term cut is also used to designate the space originally occupied by the removed material. A cross-section of a cut for a railroad is shown in Fig. 1 (*a*) and one for a highway in (*b*).

3. Fill.—The bank of earth, rock, or other material constructed above the natural surface of the ground for the support of a road, or the space occupied by such material, is known as a fill or an embankment. A cross-section of the fill for a railroad is shown in Fig. 2 (*a*) and one for a highway in (*b*).

4. Use of Material.—Usually, the material taken from a cut is employed to make a fill in the vicinity. If the excavated

earth is of poor quality or in excess of the quantity needed for the fill, it is wasted or dumped in a *spoil bank*. On the other

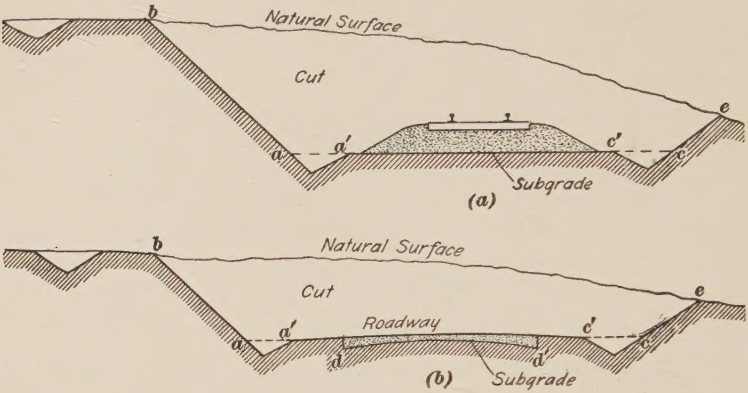


FIG. 1

hand, if the cuts do not furnish enough earth of a quality suitable for filling, the necessary material is taken from the nearest available location. A special excavation, made for this purpose, is called a *borrow pit*.

5. Subgrade.—The surface to which the cut or fill extends is called the subgrade, which means lower grade. Usually, the

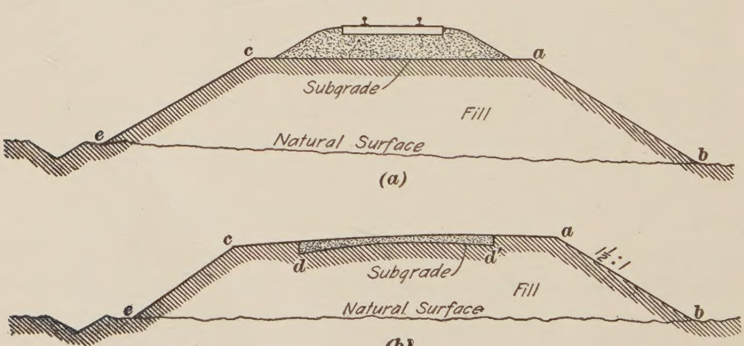


FIG. 2

subgrade is a foot to about two and one-half feet below the base of the rail in a railroad, as shown in Figs. 1 (*a*) and 2 (*a*),

and several inches below the finished wearing surface of a highway, as indicated in Figs. 1 (*b*) and 2 (*b*).

6. Roadbed.—The term roadbed is applied to the surface upon which the road material rests. The *width of roadbed* for railroads is the width *ac*, Figs. 1 (*a*) and 2 (*a*), the ditches being included. The width of roadbed for highways is the width *dd′*, Figs. 1 (*b*) and 2 (*b*).

7. Grade Line.—In the construction of a railway or highway, it is customary to fix in advance the elevation for some important line like the finished surface. This line is called the grade line. For example, in the construction of a railroad the grade line may locate the height of the subgrade, the base of rail, or the top of rail. In the same way, in the construction of a highway the grade line might represent the height of the subgrade, the finished surface, or the curb stones on either side of the road.

8. Gradient or Rate of Grade.—The inclination of the grade line to the horizontal is termed the gradient or rate of grade. Rate of grade is commonly expressed as the per cent. of rise or fall to length. Thus it may be said that the gradient or rate of grade of a roadway is +2%; this means that the elevation of the roadway increases 2 feet for each 100 feet measured horizontally. Similarly, a gradient or rate of grade of −3.23% means a fall of 3.23 feet for every 100 feet. Unless mentioned to the contrary, the gradient is stated in the direction in which the station numbers increase. For example, a rate of grade of +2% means that the elevations increase as the station numbers increase; or that one goes uphill as he follows the line.

Very frequently, especially in construction, the term *grade* is used in place of gradient or rate of grade.

9. Grade Elevation.—The elevation of the grade line at any point is called the grade elevation at that point. As previously explained, the grade line may represent different parts of the finished roadway; hence, the part for which the grade is given should be stated unless it is understood. In this Section,

as in earthwork in general, the grade elevations apply to the subgrade. Thus, the statement that the grade elevation at Sta. 26+75.8 is 771.8 means that the elevation of the subgrade at Sta. 26+75.8 is 771.8 feet above the datum.

The terms *elevation of grade* and *grade* are frequently used in place of grade elevation. As stated in the preceding article, the term grade is also used for gradient. However, since in one case the grade is an elevation and in the other a per cent., it is always easy to tell which is meant.

DIMENSIONS OF CROSS-SECTIONS

10. Side Slope.—By the side slope, or simply the slope, of a cut or fill is meant the inclination of the sides, as *ab* and *ce*, Figs. 1 and 2. A side slope is usually indicated by the rate at which the side of the cut or fill diverges from the vertical. This rate is called the *slope*, the *rate of slope*, or the *slope ratio*. Thus, a slope of 2:1 is one in which the side diverges from

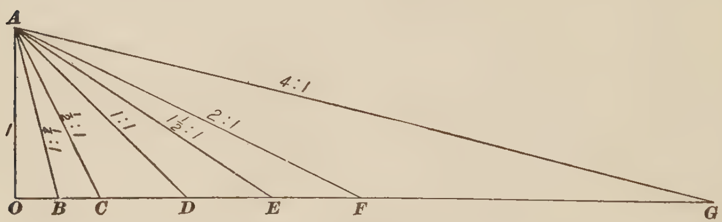


FIG. 3

the vertical at the rate of 2 units of length measured horizontally in every unit of length measured vertically. For example, in Fig. 3, which shows different rates of slope, the ratio 2:1 marked on *AF* indicates that, in the vertical distance *AO*, representing 1 foot, the horizontal distance *OF*, by which the line *AF* diverges from the vertical, is 2 feet. It will be observed that the slope of a side is the tangent of the angle that the side makes with the vertical. Thus, the slope of *AC* is equal to $\tan CAO = \frac{CO}{AO} = \frac{1}{2}:1 = .5$.

11. Slope Ratio in Cuts.—Very hard and firm rock may sometimes be safely cut out so as to leave a vertical side. How-

ever, rock that appears very hard and firm when first excavated often contains seams which may be opened by the action of frost, and as a result large pieces may break off. Therefore, care should be taken to dislodge all loose masses of rock. This will practically mean that the sides will have an average slope of about $\frac{1}{4}$:1. For weaker materials, the slopes must be flatter. For firm earth or gravel, a slope of 1:1 may be permissible for cuts, but a slope of $1\frac{1}{2}$:1 is commonly adopted, especially where the ground is liable to be washed away. If the soil is soft and treacherous, it may be necessary to make the slope as flat as 4:1. The following slope ratios have been adopted by the American Railway Engineering Association: for earth, $1\frac{1}{2}$:1; for loose rock, $\frac{1}{2}$:1; and for solid rock, $\frac{1}{4}$:1.

12. Slope Ratio in Fills.—The slope of a fill depends on the character of the material. When a fill is made with earth, a slope ratio of $1\frac{1}{2}$:1 is generally used, but flatter slopes are occasionally employed. When a fill is made from the material taken from a rock cut, the slope ratio is usually between 1:1 and $1\frac{1}{2}$:1. In side-hill work, where a slope ratio of $1\frac{1}{2}$:1, or even 1:1, might require a very long slope, it may be advisable to construct a retaining wall to lessen the amount of the fill.

13. Width of Embankments and Excavations for Railroads.—The width of roadbed required for a standard-gauge single-track railroad on a fill may be estimated as follows: At the ends of the ties, which are about $8\frac{1}{2}$ feet long, the ballast slopes down to the subgrade. The minimum extra width required for this purpose varies with the kind of ballast used, but is about 2 feet at each end of the tie. Usually, the embankment extends about 2 feet beyond the ballast on each side. The minimum width of subgrade for a fill is, therefore, $8\frac{1}{2}+2\times(2+2)$ or about $16\frac{1}{2}$ feet. This width would only be used for light-traffic, cheaply-constructed roads; 18 to 20 feet is far more common, and 22 feet or more is frequently used, as the danger of an accident due to a washing out of the embankment is materially reduced by widening the roadbed.

In a cut, extra width of roadbed must be allowed for ditches, such as *aa'* and *cc'*, Fig. 1. Unless the soil is especially firm,

the sides of the ditches should have slopes of $1\frac{1}{2}:1$. If the ditches are 16 inches deep, with side slopes of $1\frac{1}{2}:1$, each ditch requires a total width at the top of 4 feet; thus 8 feet is added to the width of the cut at the elevation of subgrade on account of the ditches. When an excavation is made through rock, the side slopes of the ditches may properly be much steeper, as there is no danger of scouring during heavy rain storms; in this case the total required width may be much less than the amount just given. The great expense of cutting through solid rock requires that such economy shall be used if possible.

The customary distance between track centers is 13 feet. Hence, to the width of the roadbed for a single-track road, 13 feet should be added for each additional track.

14. Width of Roadway for Highways and Streets.—The width of roadway, or traveled way, that should be provided for highways depends on the type and amount of traffic that is expected. The minimum width of roadway for one line of traffic is generally made 8, 9, or 10 feet, the latter figure being preferred for fast motor traffic. For two lines of traffic, the

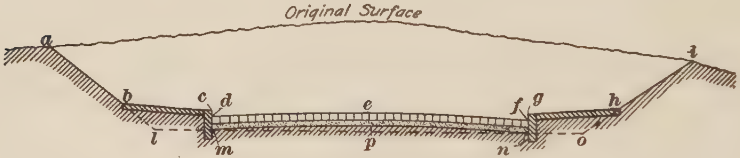


FIG. 4

usual minimum widths vary between 14 and 18 feet, the wider roadways being desirable for first-class highways provided with a hard surface.

The widths of the roadway and of the sidewalks for city streets also vary according to the expected traffic, desired appearance, and other considerations. In some cities the width of the roadway is arbitrarily made three-fifths of the total width between property lines, and the rest is left for sidewalks. In residential streets, the minimum widths between curbs that are generally considered desirable are 25 feet when there are no street-car tracks, 30 feet when there is one track, and 40 feet

when there are two tracks. In business districts, larger minimum widths are required.

Typical sections for highway work in the country are shown in Figs. 1 (*b*) and 2 (*b*). The side-slope ratios for highways are about the same as those for railroads when the conditions are similar.

A representative section of a city street is shown in Fig. 4. Here the finished surface for a cut is the broken line *abcdefghi*.

15. Ditching.—Many of the troubles in maintaining the track in a railroad or the road surface in a highway are caused by the action of water. As rain falls on the roadbed of a railroad, it scours away the soil and the ballast. Also, the water retained by the subgrade of the railroad or highway freezes in winter, heaves the soil, and produces a rough surface, which becomes soft when the ground thaws out in the spring and allows the tracks or pavement to settle. Therefore, it is of the utmost importance that adequate ditches be provided to carry away quickly from the roadbed all rain water that may fall on or near it. Ditches should be constructed on both sides of the track or roadway through cuts; the bottom of the ditch should be far enough below the subgrade to drain the water from it. A ditch should also be constructed at the top of a cut as shown near *b*, Fig. 1, so as to catch all the water that comes down the natural slope above and prevent it from washing down the side of the cut. All such ditches should lead off to some watercourse, if possible, or to some point where the discharge will not cause scour. If the soil is soft and the amount of water is very large, it may be best to pave the ditch to prevent excessive scour. In city streets the water is carried in gutters, as shown at *d* and *f*, Fig. 4.

SURVEYS

SURVEYS FOR DETERMINING GRADE

16. Introduction.—The methods of handling earthwork and computing its amount are practically the same for railroad and highway construction. Hence, unless otherwise stated, the principles here presented apply to both.

17. Preliminary Steps.—The center line of the railroad or highway is established by stakes set at intervals. Usually these are located at every full station, that is, every 100 feet from the starting point, but stakes are also driven at other important points; for example, at points where there is a change in the direction of the line. On curves, it is usual to drive a stake at the beginning and at the end of the curve, and at each full station or closer, depending on the sharpness of the curve.

TABLE I

GRADING NOTES

(1) Station	(2) H.I.	(3) Rod	(4) Elev. of Surface	(5) Elev. of Subgrade	(6) Center Depth	(7) Remarks
BM 27			98.23			N.W. cor. mon. at S. E. Cor. of Ash and Clay
	106.38	+8.15				
	(106.4)					
90		6.4	100.0	100.0	0	At grade
91		3.2	103.2	100.8	C 2.4	
91+32		2.3	104.1	101.0	C 3.1	
92		6.8	99.6	101.5	F 1.9	
92+51		9.1	97.3	101.9	F 4.6	
93		3.1	103.3	102.3	C 1.0	
93+61		3.8	102.6	102.0	C 0.6	

After the line has been staked out, the next step is to run a line of levels along the located center line to determine the elevation of the ground at each stake, and at other points where the slope of the ground changes. The grading notes for a small length of a proposed line are shown in Table I; the figures in columns 2, 3, and 4 are obtained by leveling in the usual way. It will be found much easier to compute the elevations of the surface if the values of the H.I. to the nearest tenth are written just underneath the exact elevation as in the table.

18. Profile.—After the elevations of the surface of the ground have been obtained, a profile should be plotted as shown in Fig. 5. A horizontal base line XX', assumed to represent a convenient elevation, is first drawn; in the illustration, the

chosen elevation is 80 feet above the datum. Along this line are laid out to the chosen horizontal scale the stations given in column 1 of Table I, and at each station is erected a vertical line whose length is equal to the elevation of the surface minus the elevation of the base line, measured to the chosen vertical scale. At the same time the surface elevation is written along the line representing the station. For example, Sta. 91 is plotted 100 feet from the beginning at Sta. 90. On the perpendicular at this point the ground is located at a distance above

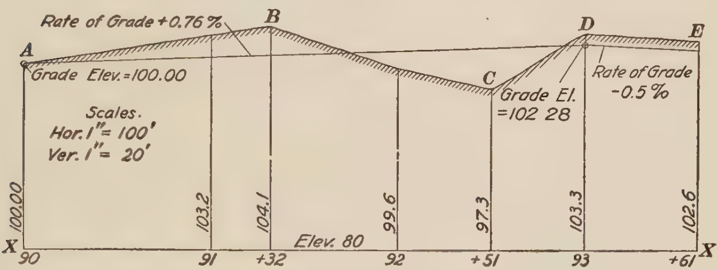


FIG. 5

the base line of 103.2−80, or 23.2, feet. On the side of this line is written the elevation of the point, 103.2. The other points on the ground are located in a similar manner and these points are connected by the free-hand line *ABCDE*.

19. Determination of Rates of Grade.—The engineer next determines the elevations to which the railroad or the highway must conform. The conditions which govern this differ somewhat for railroads and highways; they also vary with the circumstances. However, in general, there are two principal objects: (1) to build the road as cheaply as possible; (2) to make the rates of grade easy and convenient for the traffic.

The elevations of certain points are established and these points are connected by grade lines upon which are written the rates of grade. Thus, in Fig. 5, if the grade elevation at Sta. 90 is 100 and that at Sta. 93 is 102.28, the difference in elevation is $102.28-100=2.28$ and the rate of grade is $\frac{2.28}{300}=.0076$, or 0.76 per cent. Since the elevations increase as the stations

increase, the rate of grade is +0.76 per cent. The rate of grade, −0.5 per cent., beyond Sta. 93 is found in a similar manner.

20. Computation of Grade Elevations.—When the grade elevation of a certain point and the rate of grade to the next point are known, the grade elevation at the next point is readily obtained. For a positive rate of grade, the amount to be added to the elevation of the preceding point is equal to the product of the rate of grade and the distance in feet between points. If the rate of grade is negative the product is subtracted.

The calculations for determining the elevations of subgrade in column 5 of Table I may be arranged as follows:

Elevation at Sta. 90	= 100.00
Add $100 \times .0076$	.76
Elevation at Sta. 91	= 100.76 or 100.8
Add $32 \times .0076$	.24
Elevation at Sta. 91+32	= 101.00
Add $68 \times .0076$	.52
Elevation at Sta. 92	= 101.52 or 101.5
Add $51 \times .0076$	.39
Elevation at Sta. 92+51	= 101.91 or 101.9
Add $49 \times .0076$	.37
Elevation at Sta. 93	= 102.28 or 102.3
Subtract $61 \times .005$	.31
Elevation at Sta. 93+61	= 101.97 or 102.0

At every change of rate of grade, the elevation obtained in this way should always be checked with the elevation that was originally established. Thus, at Sta. 93, the computed elevation agrees with that previously established.

21. Center Depth.—The depth of the cut or fill at the center line is called the center depth. If the elevation of the surface is more than that of the grade at the same point, earth must be removed; therefore, the center depth is a cut and in the notes its value is preceded by the letter C, as in the sixth column of Table I. If the elevation of the surface at the given point

is less than that of the grade, earth must be added; in this case the center depth is a fill and in the notes its value is preceded by the letter F. Some engineers prefer to use + for cut and − for fill; thus by this method, a cut of 2.1 feet would be given as +2.1.

At Sta. 90, the elevation of the surface is 100 and the grade elevation is 100; since the two are equal, there is neither cut nor fill, and the point is said to be *at grade*.

At Sta. 91, the elevation of the surface is 103.2 and the grade elevation is 100.8; the difference is 2.4 and since the surface is above grade the center depth is C 2.4.

At Sta. 91+32 the elevation of the surface is 104.1 and the grade elevation is 101.0; the difference is 3.1 and since the surface is above grade the center depth is C 3.1.

At Sta. 92, the elevation of the surface is 99.6 and the grade elevation is 101.5; the difference is 1.9 and the center depth is F 1.9. The cut or fill at other stations is obtained in a similar way, and the values are written in column 6 of Table I.

EXAMPLES FOR PRACTICE

1. The elevations of the surface on a profile are as follows: Sta. 3, 65.0; Sta. 4, 67.1; Sta. 5, 70.8; Sta. 5+20, 71.3; Sta. 5+80, 69.8; Sta. 6, 70.9. If the elevation of subgrade at Sta. 3 is 66.40 and the gradient is +1.3%, what are the center depths at each station?

Ans. F 1.4; F 0.6; C 1.8; C 2.0; F 0.2; C 0.6

2. The elevations of the surface of the ground on a railroad line are as follows: Sta. 31, 134.9; Sta. 32, 133.0; Sta. 32+70, 132.1; Sta. 33, 132.6; Sta. 33+55, 139.6; Sta. 34, 132.4; Sta. 35, 129.2. What are the center depths at each station when the elevation of subgrade at Sta. 31 is 133.61 and the gradient is −1.22%?

Ans. C 1.3; C 0.6; C 0.6; C 1.4; C 9.1; C 2.4; C 0.5

CROSS-SECTIONING

22. Measurements for Earthwork Computations. — The first step in calculating the volume of earthwork is the determination of the areas of cross-sections of the cut and the fill at intervals along the line. These sections should be so located that if the area is assumed to vary uniformly between them the calculated volume will agree very closely with the actual value.

Hence, the sections should be spaced more closely in rough irregular country than on smooth ground. As a general rule, sections should be taken at every 100-foot station, at every point where there is an abrupt change of line or grade, and at points where fill changes to cut or cut changes to fill. The field work involved in establishing these cross-sections is called cross-sectioning.

It is important that each cross-section be taken at right angles to the line, because this is assumed in the methods used in the computations. On curves, sections should be radial, that is, they should be run toward or away from the center of the curve.

The points of the section on which elevations are taken should likewise be so chosen that the computed area will agree closely with the actual area of the section. The elevation at the center should always be obtained, and any point in the section where there is a change from cut to fill should be located. A small number of points in a section simplifies the work, but the number should not be reduced below that necessary to give the proper accuracy.

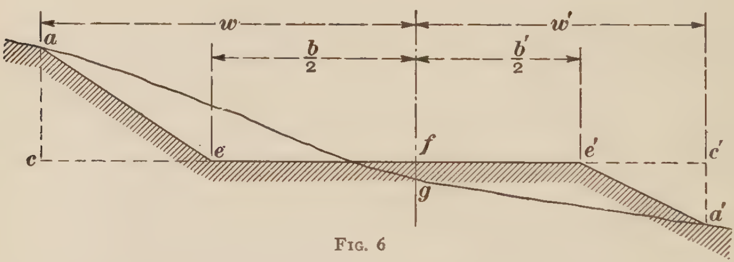


Fig. 6

23. **Slope Stakes.**—The two end points in each cross-section where the sides intersect the natural surface of the ground, as points *b* and *e* in Figs. 1 and 2, are located and marked by stakes, called slope stakes. There are two reasons for locating these points. In the first place, they are needed in order that the contractor may make the excavation or the fill to the lines intended by the engineer. Secondly, they are used in making an accurate determination of the area of a section.

Actually there is no cut or fill at a slope stake. Nevertheless, it is customary to term the vertical distance between the

ground at the slope stake and the grade line as a cut or fill, depending on whether the ground is above the grade line or below it. For example, in Fig. 6, which shows a cross-section partly in cut and partly in fill, the depth *ac* is called the cut at the slope stake at *a* and the depth *a′c′* is called the fill at the slope stake at *a′*.

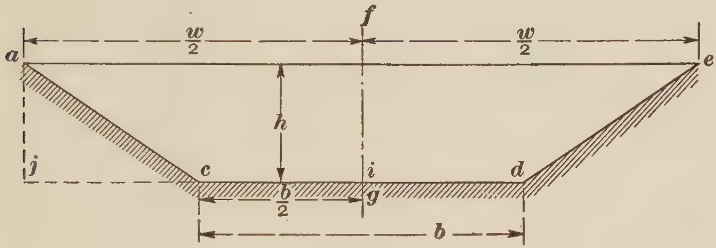


FIG. 7

24. Location of Slope Stakes When Ground Surface is Level.—When the surface of the ground is level, the cross-section is a symmetrical trapezoid, and the points at which the slope stakes are to be set can be readily located. Let *acde*, Fig. 7, represent a cross-section in level ground. The center line of the section is the line *fg* and the required positions of the slope stakes are at the points *a* and *e*. Then the distance $\frac{w}{2}$ from the center line to either slope stake is equal to $ic+cj$, or

$$\frac{w}{2}=\frac{b}{2}+sh$$

in which $\frac{w}{2}$ = distance from center line to either slope stake, in feet;

b = width of roadbed, in feet;
s = value of slope ratio;
h = center depth, in feet.

The values of b and s for the cross-section are known, and the value of h can be found in the grading notes. Hence, the distance $\frac{w}{2}$ from the center line to either slope stake can be computed by the formula of this article and laid off on the ground.

EXAMPLE.—Find the distance from the center line to either slope stake in the cross-section at Sta. 92+51 of the grading notes in Table I if the width of the roadbed is 16 feet and the slope ratio is 2:1.

SOLUTION.—Here $b=16$ ft., $s=2$, and as given in the sixth column of Table I, the center depth h is a fill of 4.6 ft. Hence,

$$\frac{w}{2}=\frac{b}{2}+sh=\frac{16}{2}+2\times 4.6=17.2 \text{ ft. Ans.}$$

25. Location of Slope Stakes When Ground Surface is Not Level.—When the surface of the ground at the cross-section is not level, as in Fig. 8, the distances from the center line to the left and right slope stakes are not equal, and their values are as given by the following formulas:

$$w_l=\frac{b}{2}+sh_l \qquad (1)$$

$$w_r=\frac{b}{2}+sh_r \qquad (2)$$

in which w_l = distance from center line to left-hand slope stake, in feet;
w_r = distance from center line to right-hand slope stake, in feet;
b = width of roadbed, in feet;
s = value of slope ratio;
h_l = cut or fill at left-hand slope stake, in feet;
h_r = cut or fill at right-hand slope stake, in feet.

However, the values of h_l and h_r are not known, and the slope stakes are usually located by trial. After the cut or fill at the center has been determined, h_l or h_r is estimated according to the slope of the ground, and the corresponding distance w_l or w_r is computed by formula 1 or 2. The computed distance is then laid off on the ground, the rod is set at the point so located, and the cut or fill at that point is determined. If the observed cut or fill agrees with the assumed value, the point is one at which the slope stake should be set. However, if there is a disagreement between the two values, the information obtained is utilized in making another assumption of the cut or fill at the slope stake; the distance w_l or w_r is once more computed and laid off on the ground; and the cut or fill at the new

trial point is determined and compared with the assumed value. The process is continued until a point is found for which the cut or fill agrees with the previously assumed value.

26. After some practice, two or three trial points will be adequate in each case to locate a slope stake with sufficient accuracy, except in difficult country where three or four trials might be required. A great deal depends on the skill and judgment of the man conducting the work.

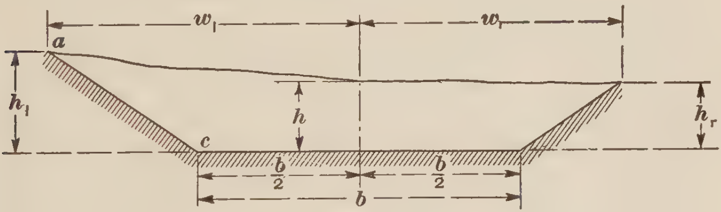


FIG. 8

As an illustration, suppose that it is required to set a slope stake at the point *a*, Fig. 8, when $\frac{b}{2}$ is 10 feet and the slope is $1\frac{1}{2}$ horizontal to 1 vertical. The height of the instrument is 130.2 feet, the grade elevation, or the height of *c*, is 123.5, and the center depth is a cut of 3.0 feet.

From an inspection of the ground, it is estimated that the cut at the slope stake is 1 foot more than at the center, or 4 feet. Then, by formula 1 of the preceding article, the corresponding distance $w_l = \frac{b}{2} + sh_l = 10 + 1\frac{1}{2} \times 4 = 16$ feet. A measurement of 16 feet is made to the left of the center and the rod reading at this point is taken. If the rod reading is 2.0, the elevation of the surface is $130.2 - 2 = 128.2$, and the cut is $128.2 - 123.5 = 4.7$. This does not agree with the assumption of 4 feet and another trial is necessary.

It is evident that in making the next trial the assumed value $h_l = 4$ feet must be increased. In turn, the corresponding value of w_l will be greater than 16 feet, and since the ground slopes upwards from the center line, the value of h_l will be greater than 4.7 feet. Let it, therefore, be assumed that $h_l = 5$ feet.

Then $w_l = 10 + 1\frac{1}{2} \times 5 = 17.5$ feet. At 17.5 feet out the rod reading is 1.4, the elevation of the surface is 128.8, and the cut is 5.3. This does not quite agree with the assumption of 5 feet and a third trial is necessary.

Let it next be assumed that h_l is 5.6 feet. Then w_l is $10 + 1\frac{1}{2} \times 5.6 = 18.4$ feet. At 18.4 feet out the rod reading is 1.1, the elevation of the surface is 129.1, and the cut is 5.6 as assumed. Hence, a slope stake should be set at this point.

27. Practical Points.—In performing the field work of setting slope stakes, a considerable saving may be effected by first finding the rod reading for grade elevation, called *grade rod,* which is the difference between the H.I. and grade elevation. For example, in the preceding article, the rod reading for a point at grade elevation, or the grade rod, is $130.2 - 123.5 = 6.7$. A rod reading less than the grade rod represents a cut equal to the difference; a rod reading greater than the grade rod indicates a fill equal to the difference. Thus, in the first trial, if the rod reading is 2.0, the cut is $6.7 - 2.0 = 4.7$ feet. The saving in time and mental effort is such that this method is usually adopted in the field.

Practice in this work will enable the engineer to perform mentally the operations for determining the location of the slope stakes. However, the final trial should be entered in the note book, both the cut or fill and the distance out being recorded. The stake should be left projecting and should be driven at an inclination with its top away from the center line; the cut or fill should then be written clearly on the side of the stake that faces the center line, and the station number on the back.

28. Notes for Cross-Section Work.—In Table II are shown typical cross-section notes.

The first column contains the stations, usually increasing toward the top of the page. In the second column are recorded the elevations of the various stations at the center line, copied from the grading notes. The third column contains the elevations of the subgrade of the railroad. On the remainder of the page are given the cross-section notes. A sketch of the section

at Sta. 131 is shown in Fig. 6; other sections are somewhat similar.

The cut or fill at each point of the section to the left or to the right of the center is recorded by drawing a short line and writing above the line the cut or fill and below the distance from the center. Thus $\frac{\text{C } 7.5}{6.0}$ means that a cut of 7.5 feet is at a point 6.0 feet out from the center; such a notation is called a *cross-section fraction.* Some engineers prefer to give the

TABLE II

CROSS-SECTION NOTES

Sta.	Elevation of Surface at Center	Elevation of Subgrade	Cross-Section		
			Left	Center	Right
132	124.0	124.2	$\frac{\text{C } 4.0}{16.0}$ $\frac{\text{C } 2.3}{10.0}$ $\frac{0}{2.2}$	F 0.2	$\frac{\text{F } 2.1}{8.0}$ $\frac{\text{F } 4.1}{16.2}$
131	123.0	123.5	$\frac{\text{C } 5.6}{18.4}$ $\frac{0}{3.2}$	F 0.5	$\frac{\text{F } 3.0}{14.0}$
130	121.3	122.8	$\frac{\text{C } 6.7}{20.0}$ $\frac{\text{C } 4.3}{15.0}$ $\frac{0}{4.5}$	F 1.5	$\frac{\text{F } 2.4}{10.0}$ $\frac{\text{F } 3.6}{15.2}$

distance above the line and the cut or fill below; for example, they would record the information just given by writing $\frac{6.0}{\text{C } 7.5}$.

Directly under the heading Center in Table II are the center depths, each being equal to the difference between the elevations in the second and third columns; as the subgrade is higher than the ground in each case, all these center depths are fills and are marked F. Under the heading Left are recorded in the same order as the arrangement in the field the cuts or fills and distances at the left, when one stands looking in the direction in which the stations increase. Under the heading Right are similar records for points to the right of the line. If now the recorder looks along the line in the direction in which the stations increase and holds his notes in front of him, the positions

of the cross-section fractions will correspond roughly with the positions of the points on which the levels were taken.

Just to the left of the center depths are recorded cross-section fractions for which the upper figure is zero. This means that neither cut nor fill is required at the given points; in other words, the points are already at grade. The cross-section fractions at the extreme right and left are for slope stakes. The width of the roadbed is taken as 20 feet in cuts and 16 feet in fills; the slopes are $1\frac{1}{2}:1$ for cuts and $2:1$ for fills.

EARTHWORK COMPUTATIONS

COMPUTATION OF VOLUME

PRISMOIDS

29. Accuracy of Computations and Measurements.—The surface of the earth is usually rough and irregular. Since it is not practicable to take an elevation at every slight change in the slope of the ground, the volume of excavation or fill necessary to bring the surface to grade is determined only approximately. The usual practice is to locate the cross-sections and to select the points in each cross-section on which levels are taken so that the average surface is represented. If the surface of the earth is assumed to slope uniformly between the cross-sections and between the points in each cross-section, and the computations are based on this assumption, the result obtained should agree closely with the actual volume.

From this description of the process of taking levels, it will be seen that extreme accuracy is a waste of time. Distances from the center line and elevations are usually read only to the nearest tenth of a foot; special sections are located a whole number of feet from a 100-foot station; and volumes are computed only to the nearest cubic yard. In ordinary work, the slide rule will give results that are accurate enough.

The method to be followed varies with the accuracy demanded and the roughness of the surface. For example, in the preliminary estimate of cost, where only an approximation is desired, volumes are sometimes computed from the elevations on the center line on the assumption that the ground at each

section is level; this gives fairly close results except on side-hill work. On the other hand, great care should be taken in determining the amount of excavation and fill when the contractor is to be paid according to the estimate. A cross-section should be measured at every point where the longitudinal slope changes enough to affect the volume of earthwork, and elevations should be taken at every point in the cross-section where the transverse slope changes enough to affect the area appreciably.

30. **Average-End-Area Method.**—Let A_1 and A_2 represent the areas in square feet of two adjacent cross-sections of earthwork, and l the distance in feet between the sections. Then the approximate volume in cubic feet of the prismoid between the two sections, as computed by the average-end-area method, is $\frac{l}{2}(A_1+A_2)$. If V_1 represents the approximate volume in cubic yards, $V_1=\frac{l}{2\times 27}(A_1+A_2)$, or

$$V_1=\frac{l}{54}(A_1+A_2)$$

The volume of a prismoid of earthwork obtained in this way is here called the *approximate volume* because it differs somewhat from the more correct value obtained by the prismoidal formula.

EXAMPLE.—A prismoid of earthwork has two rectangular bases 50 feet apart; one is 40 feet wide and 3 feet high and the other is 20 feet wide and 2 feet high. Find the volume of the prismoid by the average-end-area method.

SOLUTION.—Here, $l=50$ ft., $A_1=40\times 3=120$ sq. ft., and $A_2=20\times 2=40$ sq. ft. Then, $V_1=\frac{l}{54}(A_1+A_2)=\frac{50}{54}\times(120+40)=148$ cu. yd. Ans.

31. **Prismoidal Formula.**—The volume in cubic feet of a prismoid of earthwork between two sections, as computed by the prismoidal formula, is $\frac{l}{6}(A_1+4\,A_m+A_2)$. If V represents the volume in cubic yards, $V=\frac{l}{6\times 27}(A_1+4\,A_m+A_2)$, or

$$V=\frac{l}{162}(A_1+4\,A_m+A_2)$$

in which l = length between sections, in feet;
A_1 = area of first section, in square feet;
A_m = area of middle section halfway between first and last sections, in square feet;
A_2 = area of last section, in square feet.

EXAMPLE.—Solve the example of the preceding article by the use of the prismoidal formula.

SOLUTION.—It is first necessary to compute the value of the area of the middle section, each dimension of which is an average of the corresponding dimensions of the two end sections. Thus, the width is $\frac{40+20}{2}$ $=30$ ft. and the height is $\frac{3+2}{2}=2.5$ ft. Then, $A_m=30\times2.5=75$ sq. ft. and, as in the preceding article, $l=50$ ft., $A_1=120$ sq. ft., and $A_2=40$ sq. ft. Hence,

$$V=\frac{l}{162}(A_1+4\,A_m+A_2)=\frac{50}{162}(120+4\times75+40)$$
$$=142 \text{ cu. yd. Ans.}$$

32. Length of Prismoid of Earthwork.—The bases of a prismoid in earthwork are cross-sections like *mpqt*, Fig. 9. The

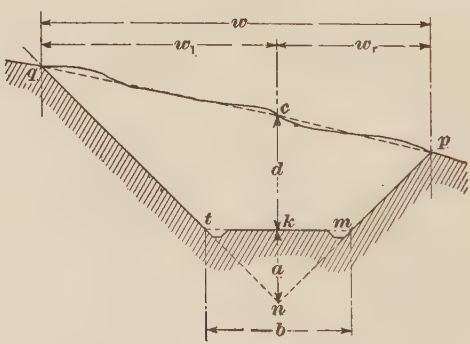


FIG. 9

length of any prismoid is equal to the distance between the cross-sections; this is usually 100 feet, unless the surface of the ground is especially rough and irregular, when it becomes necessary to take sections at intervals of less than 100 feet.

33. Method of Calculation.—The determination of the volume by the average-end-area method, or by the formula of Art. 30, usually gives fairly close results. However, the pris-

moidal formula of Art. 31 should be used for all accurate work. In order to apply the prismoidal formula, it is necessary to determine the area, A_m, of the middle section of the prismoid. This may be done by averaging the corresponding dimensions of the two ends and computing the area of the resulting figure, as in the example of Art. 31. A much simpler method, however, is to compute the approximate volume of the prismoid by

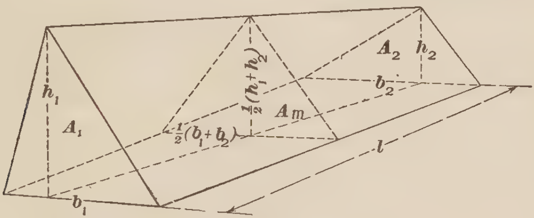


FIG. 10

the formula of Art. 30 and then, if necessary, to apply a correction to the result so obtained. This correction, called the *prismoidal correction,* is the difference between the volume V_1 computed by the formula of Art. 30 and the volume V computed by the formula of Art. 31; the result obtained by making this correction is the same as would have been calculated by a direct application of the prismoidal formula. The amount of earthwork computed by this method is therefore the *corrected volume.*

34. Volume and Prismoidal Correction for Triangular Prismoids.—A triangular prismoid, Fig. 10, is a prismoid in which the bases and all sections parallel to them are triangles. If the base and altitude of one end section are denoted by b_1 and h_1, respectively, the base and altitude of the other end section by b_2 and h_2, and the corresponding areas by A_1 and A_2, the following equations may be written:

$$A_1=\tfrac{1}{2}b_1h_1,\ A_2=\tfrac{1}{2}b_2h_2$$

The base and the altitude of the middle section, whose area is denoted by A_m are, respectively, $\frac{1}{2}(b_1+b_2)$ and $\frac{1}{2}(h_1+h_2)$. Therefore,

$$A_m=\frac{1}{2}\times\frac{b_1+b_2}{2}\times\frac{h_1+h_2}{2}=\frac{1}{4}\times\frac{(b_1+b_2)\ (h_1+h_2)}{2}$$

Let the prismoidal correction, or the difference $V-V_1$, be denoted by C. This correction is to be added *algebraically* to V_1 in order to obtain V. If the values of A_1, A_2, and A_m are substituted in the formula of Art. 31,

$$V=\frac{l}{162}(A_1+4\,A_m+A_2)$$

$$=\frac{l}{162}[\tfrac{1}{2}b_1h_1+\tfrac{1}{2}(b_1+b_2)(h_1+h_2)+\tfrac{1}{2}b_2h_2]$$

$$=\frac{l}{324}(2b_1h_1+2b_2h_2+b_1h_2+b_2h_1)$$

From the formula of Art. 30,

$$V_1=\frac{l}{54}(A_1+A_2)=\frac{l}{54}(\tfrac{1}{2}b_1h_1+\tfrac{1}{2}b_2h_2)$$

$$=\frac{l}{108}(b_1h_1+b_2h_2)=\frac{l}{324}(3b_1h_1+3b_2h_2)$$

Therefore,

$$C=V-V_1$$

$$=\frac{l}{324}[2b_1h_1+2b_2h_2+b_1h_2+b_2h_1-(3b_1h_1+3b_2h_2)]$$

$$=\frac{l}{324}(b_1h_2-b_1h_1+b_2h_1-b_2h_2)$$

$$=\frac{l}{324}[b_1(h_2-h_1)-b_2(h_2-h_1)]$$

and the formula for the prismoidal correction may be reduced to

$$C=\frac{l}{324}(b_1-b_2)(h_2-h_1) \qquad (1)$$

Also,

$$V=V_1+C \qquad (2)$$

In formulas 1 and 2

C = prismoidal correction for the length l, in cubic yards;
l = length between cross-sections, in feet;
b_1 = base of first cross-section, in feet;
b_2 = base of second cross-section, in feet;
h_2 = altitude of second cross-section, in feet;
h_1 = altitude of first cross-section, in feet;
V = corrected volume for the length l, in cubic yards;
V_1 = approximate volume for the length l, in cubic yards.

It should always be remembered that C is to be added algebraically to V_1. If C is negative, the approximate volume V_1 is too large and must be decreased; if C is positive, the approximate volume V_1 is too small and must be increased.

A study of the second member of formula 1 shows that, if either the bases or the altitudes of the two end sections are equal, one of the factors (b_1-b_2) or (h_2-h_1) is zero, and therefore the correction is zero. It shows also that, when one of these factors is small, the correction is a correspondingly small quantity; when both factors are small, the correction is very small and may often be ignored. When, as is usually the case, the base and altitude at one section are both smaller or both larger than the base and altitude at the other section, the correction is negative. Thus, if b_2 is less than b_1 and h_2 is less than h_1, then b_1-b_2 is positive and h_2-h_1 is negative, and therefore C is negative. When C is negative, V_1 is greater than the true volume V; in other words, the method of averaging end areas usually gives a result that is too large. When the difference in the bases and altitudes is very great, the correction is very large, and V_1 is greatly in error.

Example.—The dimensions of the ends of a triangular prismoid are: $b_1=18$ feet, $h_1=8$ feet, $b_2=12$ feet, and $h_2=9$ feet. Find the volume of this prismoid, in cubic yards, if the length of the prismoid is 100 feet.

Solution.—The areas of the ends are:

$$A_1=\tfrac{1}{2}\times 18\times 8=72 \text{ sq. ft.}$$
$$A_2=\tfrac{1}{2}\times 12\times 9=54 \text{ sq. ft.}$$

The approximate volume in cu. yd. is determined by substituting these values in the formula of Art. 30, or

$$V_1=\tfrac{100}{54}\times(72+54)=233 \text{ cu. yd.}$$

By formula 1, the prismoidal correction is

$$C=\tfrac{100}{324}\times(18-12)\times(9-8)=2 \text{ cu. yd.}$$

Therefore, by formula 2, the corrected volume is

$$V=233+2=235 \text{ cu. yd. Ans.}$$

EXAMPLES FOR PRACTICE

Note.—Answers are given to the nearest cubic yard.

1. If the average-end-area method is used, what is the volume of excavation in a prismoid with two rectangular ends 45 feet apart if one

end is 28 feet wide and 1.5 feet high, and the other is 28 feet wide and 3.0 feet high. Ans. $V_1=105$ cu. yd.

2. If the prismoidal formula of Art. 31 is used, what is the volume of excavation in a prismoid with two rectangular ends 100 feet apart if one end is 60 feet wide and 6 feet high, and the other is 36 feet wide and $1\frac{1}{2}$ feet high? Ans. $V=700$ cu. yd.

3. The dimensions of the ends of a triangular prismoid are $b_1=20$ feet, $h_1=10$ feet, $b_2=10$ feet, and $h_2=5$ feet. Find the corrected volume of the prismoid in cubic yards if the length of the prismoid is 100 feet. Ans. $V=216$ cu. yd.

THREE-LEVEL SECTIONS

35. Introduction.—When the surface of the ground is fairly regular, it is accurate enough to determine at each cross-section the center depth, the distances from the center line to the two slope stakes, and the cut or fill at each slope stake. It is then assumed in the computations that the ground slopes uniformly between the center and each slope stake. Thus, in Fig. 9, the straight lines *cp* and *cq* are assumed to represent the surface of the ground. When this method is used, the sections are called three-level sections.

It is customary to figure sections to certain convenient lines and then add or subtract for any irregularities. In railroad work, the usual irregularities are those due to the ditches in a cut. For example, in Fig. 9, the area *mpqt* is first determined and then the areas of the ditches are added. In highway work, as in Fig. 4, the area in cut is usually computed to the broken line *aloi*, and then the areas *blmc*, *hgno*, and *mnp* are subtracted to get the actual area.

36. Area of Three-Level Section.—In the formulas for earthwork calculations, as shown in Fig. 9, b represents the width *mt* of the roadbed; a, the distance *kn* from the roadbed to the intersection of the two side-slope lines; d, the center depth *ck*; and w_l and w_r, the horizontal distances from the center to the left-hand and the right-hand slope stakes, respectively. Also, the value of the slope ratio is denoted by s. The triangle *mnt* below the roadbed, whose altitude is a and whose base is b, is called the *grade triangle;* its area equals $\frac{1}{2}ab$. The altitude a can be found by the relation

$$a = kn = \frac{km}{\tan mnk} = \frac{\frac{1}{2}b}{s}$$

Hence,
$$a = \frac{b}{2s} \qquad (1)$$

If the area of the grade triangle *mnt* is added to the area of the given section *mpqt*, the area obtained is that of the triangle *npq*, which is equal to the sum of the areas of the two triangles *cnq* and *cnp*. The triangle *cnq* has the base $a+d$, the altitude w_l, and the area $\frac{1}{2}(a+d)w_l$. The triangle *cnp* has the same base $a+d$, the altitude w_r, and the area $\frac{1}{2}(a+d)w_r$. Therefore, the area of *npq* is

$$\tfrac{1}{2}(a+d)w_l + \tfrac{1}{2}(a+d)w_r = \tfrac{1}{2}(a+d)(w_l+w_r)$$

The area of the section *mpqt* is equal to the area of *npq* minus the area of the grade triangle *mnt*. If the area of the section *mpqt* is denoted by A, then

$$A = \tfrac{1}{2}(a+d)(w_l+w_r) - \tfrac{1}{2}ab$$

or
$$A = \tfrac{1}{2}(a+d)w - \tfrac{1}{2}ab \qquad (2)$$

in which $w_l + w_r$ is denoted by w.

37. Prismoidal Correction.—If the dimensions of the two cross-sections that form the ends of a prismoid of earthwork are very nearly equal, the method of average end areas will give a sufficiently accurate value of the volume of the prismoid; but, if the two sections differ considerably, the approximate volume V_1 should be adjusted by applying the prismoidal correction. For the three-level section this correction is computed as follows:

For the triangular prismoid to the left of the center line, the ends of which are triangles like *cnq*, Fig. 9, the prismoidal correction is, by formula 1, Art. 34,

$$\frac{l}{324}(w_l - w'_l)[(d'+a)-(d+a)] = \frac{l}{324}(w_l - w'_l)(d'-d)$$

in which w_l and d are the dimensions shown in Fig. 9 for one end and w'_l and d' represent, respectively, the same distances for the other end.

Similarly, for the triangular prismoid to the right of the center line, the ends of which are triangles like *cnp*, the prismoidal correction is

$$\frac{l}{324}(w_r - w'_r)[(d'+a)-(d+a)] = \frac{l}{324}(w_r - w'_r)(d'-d)$$

in which w_r is as shown in Fig. 9 for one end, and w'_r represents the same quantity for the other end.

Since the base and altitude of the triangular prismoid whose section is the grade triangle are constant, $(b_1 - b_2)$ and $(h_2 - h_1)$ in formula 1, Art. 34, are each zero and the correction is zero. Hence, the correction for the actual area of the three-level section is the same as the correction for the area *npq*, which in turn is the same as the sum of the corrections for the areas *cnq* and *cnp*. This sum is

$$\frac{l}{324}(w_l - w'_l)(d'-d) + \frac{l}{324}(w_r - w'_r)(d'-d)$$
$$= \frac{l}{324}(w_l + w_r - w'_l - w'_r)(d'-d)$$

Hence, if C is the prismoidal correction in cubic yards,

$$C = \frac{l}{324}(w - w')(d'-d)$$

EXAMPLE.—Find the corrected volume of fill included between Stas. 29 and 29+60, for which the cross-section notes are as follows:

Station	Left	Center	Right
29+60	$\frac{\text{F } 4.8}{17.2}$	F 2.6	$\frac{\text{F } 2.0}{13.0}$
29	$\frac{\text{F } 6.0}{19.0}$	F 4.3	$\frac{\text{F } 2.6}{13.9}$

The roadbed is 20 feet wide and the side slopes are 1½ horizontal to 1 vertical.

SOLUTION.—For Sta. 29, $a = \frac{b}{2s} = \frac{20}{2 \times 1.5} = 6.7$ ft.; $d = 4.3$ ft., $w = 19.0 + 13.9 = 32.9$ ft., $b = 20$ ft., and by formula 2, Art. 36, the area is

$$A_1 = \tfrac{1}{2}(a+d)w - \tfrac{1}{2}ab = \tfrac{1}{2} \times (6.7+4.3) \times 32.9 - \tfrac{1}{2} \times 6.7 \times 20 = 114.0 \text{ sq. ft.}$$

For Sta. 29+60, $a=6.7$ ft. as before, $d=2.6$ ft., $w=17.2+13.0=30.2$ ft., $b=20$ ft., and the area is

$$A_2=\tfrac{1}{2}\times(6.7+2.6)\times 30.2-\tfrac{1}{2}\times 6.7\times 20=73.4 \text{ sq. ft.}$$

By the formula of Art. 30,

$$V_1=\frac{l}{54}(A_1+A_2)=\tfrac{60}{54}\times(114+73.4)=208 \text{ cu. yd.}$$

The prismoidal correction is

$$C=\frac{l}{324}(w-w')(d'-d)=\tfrac{60}{324}\times(32.9-30.2)\times(2.6-4.3)=-1 \text{ cu. yd.}$$

and the corrected volume is

$$V=208-1=207 \text{ cu. yd.} \quad \text{Ans.}$$

38. Tabulation of Computations.—A convenient form for tabulating earthwork computations for three-level sections is given in Table III. The specified width of roadbed and the side slopes are recorded below the table. From these values are computed the altitude a and the area of the grade triangle. In columns 1, 2, 3, and 4 are presented the cross-section notes, which are copied from the field book.

The fifth column contains the values of $a+d$, the center depth d being recorded in the third column. For example, for Sta. 22, $a+d$ is $7.3+6.2=13.5$ feet.

The sixth column contains the values of w, the extreme width of the section, which are obtained by adding the distances below the fraction line in columns 2 and 4. For Sta. 22, $w=16.1+30.2=46.3$ feet.

Each number in the seventh column is the area of the section including the grade triangle and is one-half of the product of the numbers in columns 5 and 6. For Sta. 22, $\tfrac{1}{2}(a+d)w=\tfrac{1}{2}\times 13.5\times 46.3=312.5$ square feet.

By formula 2, Art. 36, the area of a three-level section is $\tfrac{1}{2}(a+d)w-\tfrac{1}{2}ab$. If $\tfrac{1}{2}ab$, or 80.7, as given below Table III, is subtracted from each value in column 7, the corresponding result in column 8 is the area of that section. Thus, the area of the section at Sta. 22 is $312.5-80.7=231.8$ square feet.

By formula 1, Art. 30, the approximate volume between any two consecutive sections is equal to $\frac{l}{54}(A_1+A_2)$. For the pris-

TABLE III

EARTHWORK CALCULATIONS FOR THREE-LEVEL SECTIONS

Station (1)	Cross-Section Left (2)	Cross-Section Center (3)	Cross-Section Right (4)	$a+d$ Ft. (5)	w Ft. (6)	$\frac{1}{2}(a+d)w$ (7)	Area Sq. Ft. (8)	Approx. Volume Cu. Yd. (9)	$w-w'$ Ft. (10)	$d'-d$ Ft. (11)	Prismoidal Correction Cu. Yd. (12)	Corrected Volume Cu. Yd. (13)
25	$\frac{C\ 0.6}{11.9}$	C 2.4	$\frac{C\ 4.7}{18.0}$	9.7	29.9	145.0	64.3					
								425	+18.1	−5.7	−21	404
24+35	$\frac{C\ 5.9}{19.9}$	C 8.1	$\frac{C\ 11.4}{28.1}$	15.4	48.0	369.6	288.9					
								531	+16.0	−3.7	− 6	525
24	$\frac{C\ 8.8}{24.2}$	C 11.8	$\frac{C\ 19.2}{39.8}$	19.1	64.0	611.2	530.5					
								1,600	−14.4	+2.4	−11	1,589
23	$\frac{C\ 4.8}{18.2}$	C 9.4	$\frac{C\ 13.6}{31.4}$	16.7	49.6	414.2	333.5					
								1,047	− 3.3	+3.2	− 3	1,044
22	$\frac{C\ 3.4}{16.1}$	C 6.2	$\frac{C\ 12.8}{30.2}$	13.5	46.3	312.5	231.8					

NOTES.—

Side slopes are $1\frac{1}{2}:1$.

Width of roadbed b is 22 feet.

$a=\frac{b}{2s}=\frac{22}{3}=7.3$ feet.

Area of grade triangle is $\frac{1}{2}ab=\frac{1}{2}\times\frac{22}{3}\times 22=80.7$ square feet.

moid between Stas. 22 and 23, $A_1 = 231.8$ square feet, $A_2 = 333.5$ square feet, and $l = 100$ feet. Hence, the approximate volume is $\frac{100}{54} \times (231.8 + 333.5) = 1{,}047$ cubic yards, as given in column 9.

The remainder of the table is employed when it is desired to apply the prismoidal correction. The tenth column contains values of $w - w'$, computed from the quantities in column 6. Thus, for the prismoid between Stas. 22 and 23, w for Sta. 22 is 46.3; w', or w for Sta. 23, is 49.6; and $w - w'$ is $46.3 - 49.6$ or -3.3 feet.

In the eleventh column are given values of $d' - d$ computed from the quantities in column 3. Thus, between Stas. 22 and 23, d', or d for Sta. 23, is 9.4; d for Sta. 22 is 6.2; and $d' - d$ is $9.4 - 6.2$ or $+3.2$ feet. Care must be taken that d and w are for one cross-section and d' and w' for the other.

The twelfth column contains the value of the prismoidal correction, as computed by the formula of Art. 37. Thus, between Stas. 22 and 23,

$$C = \frac{l}{324}(w - w')(d' - d) = \frac{100}{324} \times (-3.3) \times 3.2 = -3 \text{ cubic yards}$$

Hence, 3 cubic yards is to be deducted from the volume obtained by the approximate method. The resulting corrected value is given in the thirteenth column.

If, as often happens, sections occur that are not of the three-level type, the notes are inserted in columns 1, 2, 3, and 4, but no entry is made in columns 5 and 7. The extreme width of the section is placed in column 6 and the area as determined by methods given later is placed in column 9; otherwise, the tabulation is like that in Table III.

In practice it is usual to check or verify computations, two men doing the work independently. The method followed in Table III is advantageous for this purpose because all steps are shown in the tabulation; hence, errors are easily located in case there is a difference in results.

EXAMPLES FOR PRACTICE

The following notes have been recorded for a single-track railway with a width of roadbed of 21 feet and side slopes of $1\frac{1}{2}$ horizontal to 1 vertical.

Station	Left	Center	Right
162+45	$\frac{F\,2.4}{14.1}$	F 4.6	$\frac{0}{10.5}$
162	$\frac{F\,2.1}{13.6}$	F 4.6	$\frac{F\,4.1}{16.6}$
161	$\frac{F\,4.0}{16.5}$	F 2.1	$\frac{F\,6.1}{19.6}$

1. Find the approximate volume between Stas. 161 and 162.
Ans. 356 cu. yd.

2. What is the corrected volume between Stas. 161 and 162?
Ans. 361 cu. yd.

3. Compute the approximate volume between Stas. 162 and 162+45.
Ans. 142 cu. yd.

4. Determine the prismoidal correction between Stas. 162 and 162+45.
Ans. 0

IRREGULAR SECTIONS

39. Computation of Areas.—A section for which elevations are taken at points other than the center and the slope stakes is called an irregular section. Such sections are very common in highway construction when an old road is graded. They also occur in both highway and railroad work when the surface is uneven.

A typical irregular section is shown in Fig. 11. The line *abcdef* represents the original surface, which is regarded as a series of straight lines connecting the points *a, b, c, d, e,* and *f,* where elevations are taken. The surface is to be cut down to the lines *ag*, *gh*, and *hf*; hence, it is required to find the area of the section *abcdefhg* in order to obtain the volume of excavation.

If the irregular cross-section is divided by vertical and inclined lines as shown by the dotted lines, the area of the triangle *abi* is equal to $\frac{1}{2}bi \times aj = \frac{1}{2}h_2(x_1 - x_2)$. The area of the triangle *bci* is equal to $\frac{1}{2}bi \times kc = \frac{1}{2}h_2(x_2 - x_3)$. The sum of the areas of these two triangles, which have the common base *bi*, is area $abci = \frac{1}{2}h_2(x_1 - x_2 + x_2 - x_3) = \frac{1}{2}h_2(x_1 - x_3)$.

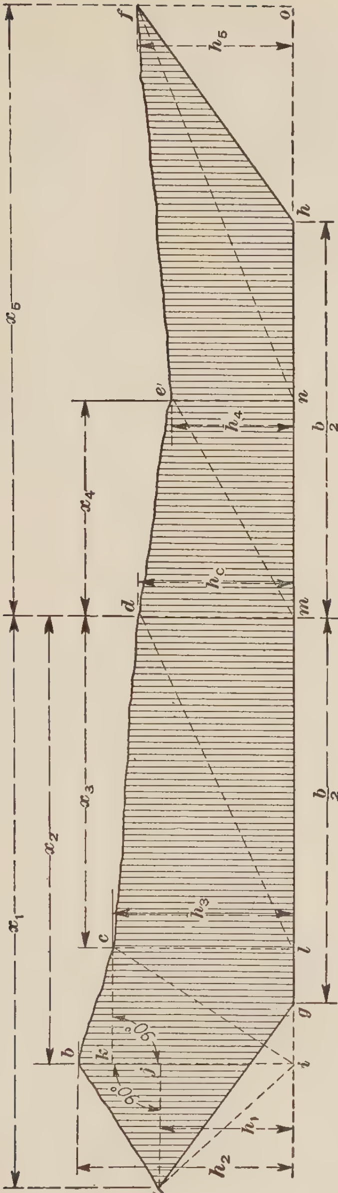

a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
x_1
x_2
x_3
x_4
x_5
h_1
h_2
h_3
h_4
h_5
h_c
$\frac{b}{2}$
$\frac{b}{2}$
90°
90°

FIG. 11

In a similar way,

$$\text{area } cdli \;= \tfrac{1}{2}cl(il+lm) = \tfrac{1}{2}h_3x_2$$
$$\text{area } deml = \tfrac{1}{2}dm(lm+mn) = \tfrac{1}{2}h_c(x_3+x_4)$$
$$\text{area } efnm = \tfrac{1}{2}en(mn+no) = \tfrac{1}{2}h_4x_5$$
$$\text{area } nhf \;= \tfrac{1}{2}nh \times fo = \tfrac{1}{2}\left(\frac{b}{2}-x_4\right)h_5$$
$$\text{area } agi \;= \tfrac{1}{2}ig \times ij = \tfrac{1}{2}\left(x_2-\frac{b}{2}\right)h_1$$

The entire area of the excavation is area *abci*+area *cdli* +area *deml*+area *efnm*+area *nhf*−area *agi*, or

$$\tfrac{1}{2}\left[h_2(x_1-x_3)+h_3x_2+h_c(x_3+x_4)+h_4x_5+h_5\left(\frac{b}{2}-x_4\right)-h_1\left(x_2-\frac{b}{2}\right)\right]$$

40. Method of Procedure.—In accordance with the explanation given in the preceding article, the area of any irregular section, either entirely in cut or entirely in fill, may be computed in the following manner:

(1) Multiply the depth at each intermediate point, including that at the center, by the horizontal distance between the adjacent points, one on each side of the intermediate point, at which elevations are taken. Each product so obtained is positive.

(2) Multiply the depth at each slope stake, or outside point, by the difference between one-half the width of the roadbed and the distance from the center to the intermediate point nearest the outside. The product will be positive if one-half of the width of the roadbed is greater than the distance to the intermediate point and negative if one-half of that width is less than the distance to the intermediate point.

(3) Add to the sum of the products in (1) the positive value or values in (2) and subtract the negative value or values in (2). The total area of the section is one-half of the result.

Example.—The following notes having been recorded at Sta. 129, it is required to find the area of the cross-section; the roadbed is 24 feet wide and the side-slope ratio is 1½:1.

Station	Left			Center	Right	
129	$\frac{C\,12.7}{31.0}$	$\frac{C\,16.0}{15.0}$	$\frac{C\,12.2}{10.5}$	C 8.3	$\frac{C\,4.1}{8.2}$	$\frac{C\,6.0}{21.0}$

SOLUTION.—The cross-section is similar to that shown in Fig. 11. Beginning on the left, the first intermediate depth is 16.0 and the horizontal distance between the two adjacent points is $31.0-10.5=20.5$. Hence, the first product is $16\times20.5=+328.0$ sq. ft.

The second intermediate depth is 12.2, the horizontal distance between the adjacent points is 15, and the product is $12.2\times15=+183.0$ sq. ft.

The third intermediate depth is 8.3, the horizontal distance between the adjacent points is $10.5+8.2=18.7$, and the product is $8.3\times18.7=+155.2$ sq. ft.

The fourth intermediate depth is 4.1, the horizontal distance between adjacent points is 21, and the product is $4.1\times21=+86.1$ sq. ft.

The depth at the left-hand slope stake is 12.7, one-half of the width of the roadbed minus the horizontal distance from the center to the intermediate point nearest to the outside is $\frac{24}{2}-15=-3$, and the product is $12.7\times(-3)=-38.1$ sq. ft.

The depth at the right-hand slope stake is 6, one-half the width of the roadbed minus the horizontal distance from the center to the intermediate depth nearest the outside is $\frac{24}{2}-8.2=+3.8$, and the product is $6\times3.8=+22.8$ sq. ft.

The total area is $\frac{1}{2}(328.0+183.0+155.2+86.1-38.1+22.8)=368.5$ sq. ft. Ans.

41. Illustrative Case.—A method of tabulating earthwork calculations for irregular sections is illustrated in Table IV; except for the calculations of areas, it is merely an extension of the method for three-level sections.

In columns 1, 2, 3, and 4 are given the stations and the corresponding cross-section notes, which are copied from the field notes. Column 5 contains the double areas for the parts of the section as computed by the method of Art. 40, and column 6 contains the total area of the section.

A convenient arrangement of the computations for the area is as given below for Sta. 128+40.

Depth		Horizontal Distance	Product
Left intermediate	$=20.4$	$46.2-19.5=26.7$	$+544.7$
Next intermediate	$=18.2$	31.0	$+564.2$
Center	$=13.2$	$19.5+13.7=33.2$	$+438.2$
Right intermediate	$=12.8$	27.6	$+353.3$
Left outside	$=22.8$	$\frac{24}{2}-31\ =-19.0$	-433.2
Right outside	$=10.4$	$\frac{24}{2}-13.7=-\ 1.7$	$-\ 17.7$
		Total	1,449.5

Area $=\frac{1}{2}\times1{,}449.5=724.8$ square feet.

The notes for Stas. 126 and 127 show that they are three-level sections. In Table IV their areas have been computed according to the rule for irregular sections, but the results may be checked by formula 2, Art. 36. Thus, for Sta. 126, $A=\frac{1}{2}(a+d)w-\frac{1}{2}ab=\frac{1}{2}\times(8+4.2)\times 41.5-\frac{1}{2}\times 8\times 24=157.2$ square feet. This result agrees closely with that obtained by considering Sta. 126 as an irregular section.

The values in the seventh column are the approximate volumes, in cubic yards, of the excavation between adjacent stations. These results are obtained by applying formula 1, Art. 30. Thus, between Stas. 128+40 and 129, the distance is 60 feet, and the approximate volume is $V_1=\frac{l}{54}(A_1+A_2)$ $=\frac{60}{54}\times(724.8+368.5)=1{,}215$ cubic yards.

The values in the eighth column are the extreme widths of the sections or the widths between slope stakes. Thus, for Sta. 129, $w=31.0+21.0=52.0$ feet.

Each value in the ninth column is equal to the difference between the extreme widths w and w' at the two adjacent sections that constitute the ends of a prismoid, w being the extreme width at the first section, for which the station number is lower, and w' being the extreme width at the second section. Thus, between Stas. 127 and 128, $w-w'=64.5-64.6=-.1$.

Each value in the tenth column is equal to the difference between the center depths of the second and first sections at the ends of the prismoid. Between Stas. 126 and 127, $d'-d=8.6-4.2=+4.4$ feet.

The values in the eleventh column for the three-level sections are the amounts of the prismoidal correction, determined by the formula of Art. 37. Between Stas. 126 and 127,

$$C=\frac{l}{324}(w-w')(d'-d)=\tfrac{100}{324}\times(-23.0)\times 4.4=-31 \text{ cu. yd.}$$

In the twelfth column are given the corrected volumes, which are obtained by applying the prismoidal corrections to the approximate volumes in column 7. The corrected volume between Stas. 126 and 127 is 1,105−31 or 1,074 cubic yards.

TA

EARTHWORK FOR

Station (1)	Cross-Section: Left (2)	Cross-Section: Center (3)	Cross-Section: Right (4)
129	$\frac{C\ 12.7}{31.0}$ $\frac{C\ 16.0}{15.0}$ $\frac{C\ 12.2}{10.5}$	C 8.3	$\frac{C\ 4.1}{8.2}$ $\frac{C\ 6.0}{21.0}$
128+40	$\frac{C\ 22.8}{46.2}$ $\frac{C\ 20.4}{31.0}$ $\frac{C\ 18.2}{19.5}$	C 13.2	$\frac{C\ 12.8}{13.7}$ $\frac{C\ 10.4}{27.6}$
128	$\frac{C\ 18.6}{39.9}$	C 10.9	$\frac{C\ 8.0}{4.2}$ $\frac{C\ 8.5}{24.7}$
127	$\frac{C\ 14.6}{33.9}$	C 8.6	$\frac{C\ 12.4}{30.6}$
126	$\frac{C\ 9.6}{26.4}$	C 4.2	$\frac{C\ 2.1}{15.1}$
Total			

Roadbed 24 feet wide in cut.
Slope ratio, $1\frac{1}{2}$:1.

V

ULAR SECTIONS

le a Ft.	Total Area Sq. Ft. (6)	Approximate Volume Cu. Yd. (7)	w (8)	$w-w'$ (9)	$d'-d$ (10)	Prismoidal Correction Cu. Yd. (11)	Corrected Volume Cu. Yd. (12)
3.0							
3.0							
5.2	368.5		52.0				
6.1							
3.1							
2.8		1,215		+21.8	−4.9	−20	1,195
4.7							
4.2							
3.2	724.8		73.8				
3.3							
3.2							
7.7		895		− 9.2	+2.3	− 3	892
0.7							
7.6	483.9		64.6				
3.2							
6.3		1,710		− .1	+2.3	0	1,710
54.7							
5.2	439.4		64.5				
8.8		1,105		−23.0	+4.4	−31	1,074
4.3							
5.2	157.4		41.5				
5.2							
		4,925				−54	4,871

94772

The true prismoidal correction for a prismoid with irregular sections as ends differs little from the correction for a prismoid for which the ends are three-level sections having the same total widths and center depths. For this reason, it is customary, in applying the prismoidal correction to ground where irregular sections are recorded, to use the formula of Art. 37. This has been done in Table IV; thus between Stas. 128+40 and 129

$$C=\frac{l}{324}(w-w')(d'-d)=\tfrac{60}{324}\times 21.8\times(-4.9)=-20 \text{ cu.yd.}$$

EXAMPLES FOR PRACTICE

The following notes have been taken for two stations on a fill for a railroad whose roadbed is 20 feet wide; the side-slope ratio is 1½:1.

Station	Left		Center	Right	
22	$\frac{F\,4.1}{16.1}$	$\frac{F\,5.0}{8.0}$	F 6.5	$\frac{F\,2.0}{8.0}$	$\frac{F\,7.5}{21.2}$
21	$\frac{F\,6.2}{19.3}$	$\frac{F\,6.0}{12.0}$	F 5.3	$\frac{F\,4.0}{10.0}$	$\frac{F\,8.9}{23.3}$

1. Compute the approximate volume of the fill between Stas. 21 and 22. Ans. 522 cu. yd.

2. Obtain the prismoidal correction for the fill between Stas. 21 and 22. Ans. +2 cu. yd.

3. What is the corrected volume of fill between Stas. 21 and 22? Ans. 524 cu. yd.

SIDE-HILL WORK

42. Introduction.—A railroad or a highway is often located on ground that has a decided slope at right angles to the line. Under these circumstances, it is quite advantageous to choose the grade so that there is cut and fill in the same section. Such an arrangement lessens the amount of earth to be moved and lowers the cost per cubic yard, because the dirt can often be shoveled directly from cut to fill. On account of this economy, earthwork of this kind, called *side-hill work,* is quite common. Sections partly in cut and partly in fill also occur in locations where the transverse slope is small; the computations of volumes for such situations follow the principles of side-hill work.

43. Areas of Sections.—A typical section in side-hill work is shown in Fig. 12, in which the original ground is represented by the line *nptv* and the graded surface by the lines *nm*, *mq*, and *qv*. It should be observed that the shoulder *m* in fill is only 8 feet from the center line, whereas the point *q* in the cut is

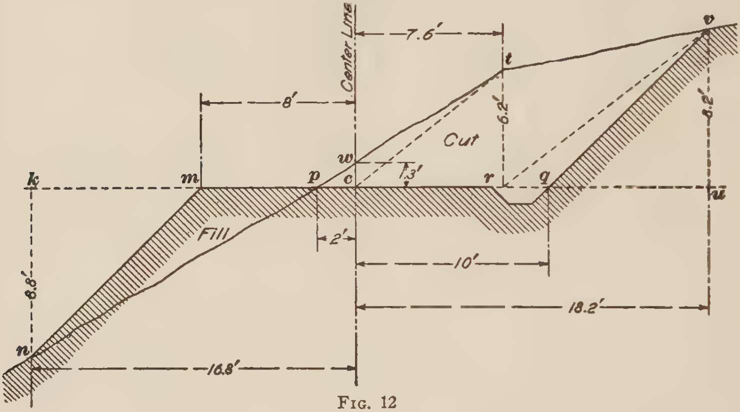


FIG. 12

10 feet from the center line. If the ground on the uphill-side had sloped uniformly between *p* and *v*, it would have been unnecessary to take an elevation at *t*. As for sections wholly in cut or in fill, the number of points at which elevations are taken depends on the topography of the ground. The areas of the cut and fill in each section are determined separately by the method best suited to the shape of the figure.

Although the amount of earthwork in side-hill construction is usually small, yet the prismoidal correction is comparatively large and should be applied to volumes obtained by the method of averaging end areas.

EXAMPLE.—Compute the area of fill and the area of cut for the section at Sta. 33, to which the following notes and Fig. 12 apply. The

Station	Left		Center	Right	
33	$\frac{\text{F } 8.8}{16.8}$	$\frac{0}{2.0}$	C 1.3	$\frac{\text{C } 6.2}{7.6}$	$\frac{\text{C } 8.2}{18.2}$

roadbed extends 8 feet from the center on the down-hill side and 10 feet from the center on the up-hill side. The side-slope ratio is 1 to 1 in fill and in cut.

SOLUTION.—The area of the fill is the area of the triangle *nmp*, whose base $mp=8-2=6$ ft., and whose altitude $kn=8.8$ ft.

The area of the fill is $\frac{1}{2}\times6\times8.8=26.4$ sq. ft. Ans.

The area of the cut is the area of the figure *pwtvq*, which equals area *pwtc*+area *ctvr*+area *rvq*.

The area of *pwtc* equals the area of the triangle *pwc* plus the area of the triangle *wtc*, or

$$\tfrac{1}{2}\times1.3\times2+\tfrac{1}{2}\times1.3\times7.6=\tfrac{1}{2}\times1.3\times9.6=6.2 \text{ sq. ft.}$$

Similarly, the area of *ctvr* is

$$\tfrac{1}{2}\times6.2\times7.6+\tfrac{1}{2}\times6.2\times(18.2-7.6)=\tfrac{1}{2}\times6.2\times18.2=56.4 \text{ sq. ft.}$$

The area of the triangle *rvq* is

$$\tfrac{1}{2}(10-7.6)\times8.2=9.8 \text{ sq. ft.}$$

The total area of the cut is $6.2+56.4+9.8=72.4$ sq. ft. Ans.

44. Volume Computations in Side-Hill Work.—The volumes of the prismoids of cut and fill are computed separately. The areas of the ends of each prismoid are first found and the volume V_1 is computed by the formula of Art. 30.

For the purpose of computing the prismoidal correction, it is sufficiently accurate to regard the ends of the prismoids as triangles, and to apply formula 1, Art. 34.

EXAMPLE.—It is required to compute from the following notes the volumes of cut and fill between Stas. 32 and 33, the roadbed being 20 feet wide in cuts and 16 feet wide in fills. The side slopes both in cut and in fill are 1 to 1.

Station	Left		Center	Right		
34	$\frac{\text{F }10.0}{18.0}$	$\frac{0}{5.0}$	C 4.0			$\frac{\text{C }9.4}{19.4}$
33	$\frac{\text{F }8.8}{16.8}$	$\frac{0}{2.0}$	C 1.3		$\frac{\text{C }6.2}{7.6}$	$\frac{\text{C }8.2}{18.2}$
32	$\frac{\text{F }11.5}{19.5}$		F 2.0	$\frac{0}{3.4}$	$\frac{\text{C }3.3}{11.6}$	$\frac{\text{C }6.0}{16.0}$

SOLUTION.—The cross-section at Sta. 32 is shown in Fig. 13, in which *abcde* represents the surface of the ground and *afghe* the finished grade.

The area of the fill equals the area of the figure *agcb* plus the area of the triangle *afg*.

Area $agcb = \frac{1}{2}\times 2\times 3.4 + \frac{1}{2}\times 2\times 19.5 = 22.9$ sq. ft.

Area $afg = \frac{1}{2}\times 8\times 11.5 = 46.0$ sq. ft.

Total area of fill is $22.9+46.0=68.9$ sq. ft.

The area of the cut equals the area of the figure *cdei* minus the area of the triangle *ehi*.

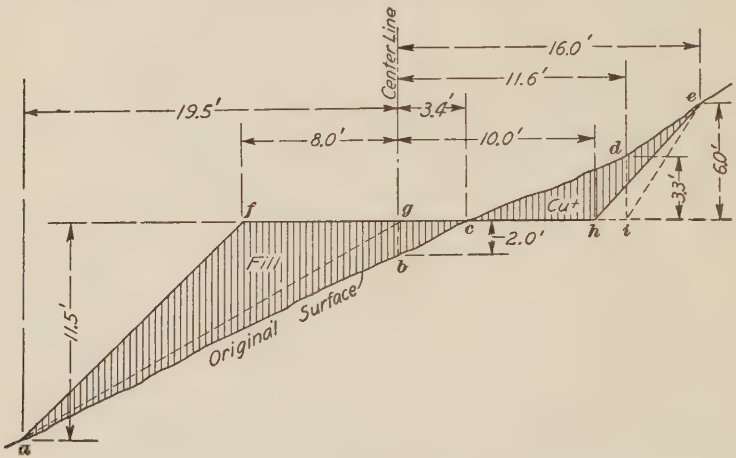


FIG. 13

Area $cdei = \frac{1}{2}\times 3.3\times(11.6-3.4) + \frac{1}{2}\times 3.3\times(16.0-11.6) = 20.8$ sq. ft.

Area $ehi = \frac{1}{2}(11.6-10.0)\times 6.0 = 4.8$ sq. ft.

Total area of cut is $20.8-4.8=16.0$ sq. ft.

In the example of the preceding article it was found that, for Sta. 33, the area of fill is 26.4 sq. ft. and the area of cut is 72.4 sq. ft. The approximate volume of fill is, therefore,

$$V_1 = \frac{l}{54}(A_1+A_2) = \frac{100}{54}\times(68.9+26.4) = 176 \text{ cu. yd.}$$

and that of cut is

$$\frac{100}{54}\times(16.0+72.4) = 164 \text{ cu. yd.}$$

The prismoidal correction is now computed by formula 1, Art. 34.

For fill, $C = \frac{100}{324}\times(11.4-6.0)\times(8.8-11.5) = -4$ cu. yd.

For cut, $C = \frac{100}{324}\times(6.6-12.0)\times(8.2-6.0) = -4$ cu. yd.

The corrected volumes are, therefore,

for fill, $V = 176-4 = 172$ cu. yd. Ans.

for cut, $V = 164-4 = 160$ cu. yd. Ans.

EXAMPLES FOR PRACTICE

In the example of Art. 44, find the volumes of the prismoids of cut and fill between Stas. 33 and 34.

$$\text{Ans.}\begin{cases}\text{For fill, } V_1 = 77;\ C = +1;\ V = 78 \text{ cu. yd.}\\ \text{For cut, } V_1 = 311;\ C = -1;\ V = 310 \text{ cu. yd.}\end{cases}$$

TRANSITION FROM CUT TO FILL

45. Cross-Sections.—The typical conditions which exist when there is a change from cut to fill are shown in Fig. 14. The line *abcde* indicates the center line on the natural surface

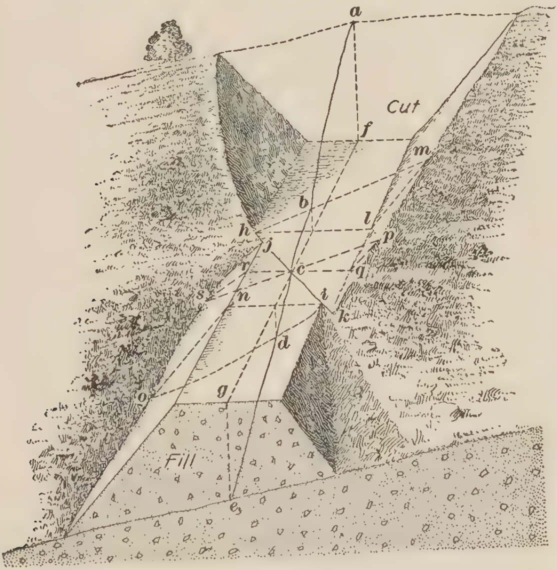


Fig. 14

of the ground and the line *fcg* is the grade line at the center of the roadbed. There is neither cut nor fill at the points *h*, *c*, and *i*; that is, the earthwork changes from cut to fill along the line *hci* and the points *h*, *c*, and *i* are *at grade*.

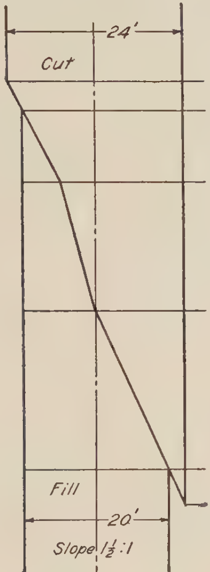
A cross-section is always taken at the point *c*. If the line *hci* of no cut and no fill is at right angles to the center line, or nearly so, other sections are taken at the nearest break in the

ground or at the nearest full station on either side. If that line is not at right angles, as it very often happens, cross-sections are taken at the point *h* where the roadbed first runs out of the cut and at the point *i* where the roadbed first attains its full width in the fill. Between *h* and *i*, a section is taken at *c*, at every decided break in the ground, and at every full station. Outside of the length between *h* and *i*, sections are taken on either side at the nearest full station or at the nearest abrupt break in the ground. Very often the width of the roadbed in cut differs from the width in fill; for example, in Fig. 14, the roadbed in cut is wider than that in fill, as it is common in railroads to allow for side ditches. Hence, the station of *j* is likely to differ from that of *h* and similarly the stations of *i* and *k* are different. In order to compute the earthwork in such a case, it is also necessary to record the stations of the points *j* and *k*.

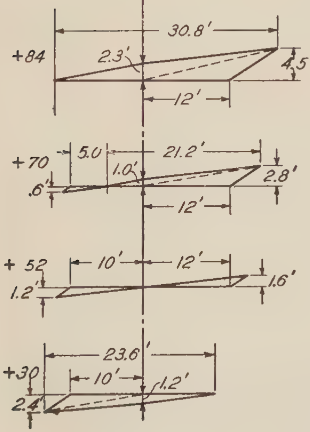
46. Methods of Computations.—The first step in computing volumes for the portion of a roadbed in which there is a change from cut to fill is to calculate the areas of the cross-sections. When the line *hci*, Fig. 14, is at right angles to the center line or nearly so, the area at *c* may be assumed to be zero and the volume between *c* and the nearest section on either side may be assumed to be a wedge. The contents of each wedge can then be computed by multiplying one-half the area of the nearest section, which is taken as a base, by the horizontal distance from the base to the point *c*.

The cross-section through the points *b* and *h* is the figure *bhlm*, which is entirely in cut; that through the point *c* is the figure *pqcrs*, which is partly in fill and partly in cut; that through the point *i* is the figure *idon*, which is entirely in fill. Then the volume of the cut between *b* and *c* may be obtained by substituting the end areas *bhlm* and *pqc* in the formula of Art. 30, and applying the prismoidal correction; and the volume of the pyramid of cut between *c* and *k* is equal to one-third of the product of the area of the base *pqc* and the horizontal distance between *c* and *k* measured along the center line. Similarly the volume of the fill between *d* and *c* can be found by substituting the end areas *dino* and *crs* in the formula of Art. 30

and applying the prismoidal correction; and the volume, in cubic feet, of the end pyramid between *c* and *j* is equal to one-



Sta.	Left	Center	Right
+84	$\frac{0}{12.0}$	C 2.3	$\frac{C\,4.5}{18.8}$
+80	$\frac{0}{10.0}$		
+70	$\frac{F\,0.6}{10.9}$ $\frac{0}{5.0}$	C 1.0	$\frac{C\,2.8}{16.2}$
+52	$\frac{F\,1.2}{11.8}$	0	$\frac{C\,1.6}{14.4}$
+30	$\frac{F\,2.4}{13.6}$	F 1.2	$\frac{0}{10.0}$
164+25			$\frac{0}{12.0}$



For Sta. 164+84

Area of cut $= \frac{1}{2}\times 2.3\times 30.8+\frac{1}{2}\times 12\times 4.5$
$=62.4$ sq. ft.

For Sta. 164+70

Area of cut $= \frac{1}{2}\times 1\times 21.2+\frac{1}{2}\times 12\times 2.8$
$=27.4$ sq. ft.
Area of fill $= \frac{1}{2}\times 5\times .6=1.5$ sq. ft.

For Sta. 164+52

Area of cut $= \frac{1}{2}\times 12\times 1.6=9.6$ sq. ft.
Area of fill $= \frac{1}{2}\times 10\times 1.2=6.0$ sq. ft.

For Sta. 164+30

Area of fill $= \frac{1}{2}\times 1.2\times 23.6+\frac{1}{2}\times 10\times 2.4$
$=26.2$ sq. ft.

FIG. 15

third of the product of the area *crs* and the horizontal distance between *c* and *j* measured along the center line.

47. Notes.—A typical set of notes, sketches, and computations of areas, for a length in which there is a change from cut to fill is shown in Fig. 15. The sketches might be omitted but they assist the work to such an extent and are so likely to lessen

TABLE V

EARTHWORK CALCULATIONS AT TRANSITION FROM CUT TO FILL

Station	Area Sq. Ft.	Approx. Volume Cu. Yd.	Prismoidal Correction Cu. Yd.	Corrected Volume Cu. Yd.
		Cut		
164+84	62.4			
		23	−1	22
164+70	27.4			
		12	0	12
164+52	9.6			
				3
164+25	0			
		Fill		
164+80	0			
				0
164+70	1.5			
		3	0	3
164+52	6.0			
		13	−1	12
164+30	26.2			

errors that it is better to employ them. In the upper left-hand corner is a plan which shows the ground under consideration. Opposite each point where a depth or a cross-section is given are shown the corresponding notes. Below the plan are given sketches for the cross-sections with the necessary dimensions marked on them. Opposite them are the computations for the areas, the methods employed being those already used in side-hill work.

48. Calculations of Volume.—The values for the determination of the volume of the cut and fill for the length whose cross-sections are given in Fig. 15, are shown in Table V. In the left-hand column are the stations and opposite them in the second column are the corresponding areas of cut.

In the third column are the approximate volumes of cut determined by applying the formula of Art. 30. Thus, between Stas. 164+70 and 164+84, $l=84-70=14$ feet, $A_1=27.4$ square feet, $A_2=62.4$ square feet, and $V_1=\frac{l}{54}(A_1+A_2)=\frac{14}{54}\times(27.4+62.4)=23$ cubic yards.

In the fourth column are the prismoidal corrections for the cut, obtained by the use of formula 1, Art. 34. Thus, between Stas. 164+70 and 164+84,

$$C=\frac{l}{324}(b_1-b_2)(h_2-h_1)=\frac{14}{324}\times(17-24)\times(4.5-2.8)$$
$$=-1 \text{ cubic yard}$$

The corrected volumes in the fifth column are obtained by applying the prismoidal corrections to the approximate volumes in column 3.

Between Stas. 164+25 and 164+52, no values are inserted in the third and fourth columns, for this portion is a pyramid and the required volume in cubic yards is computed at once by multiplying one-third of the area of the cut at Sta. 164+52 by the distance between Stas. 164+25 and 164+52, and dividing by 27.

The methods employed in calculating the volume of the fill are similar to those used in obtaining the cut.

MISCELLANEOUS PROBLEMS

49. Excavation in Compound Section.—When excavating, rock is often found underlying the earth; a section where both earth and rock must be removed is termed a compound section. If rock is encountered, the contractor exposes enough to allow the engineer to take the necessary levels; for a short time the earth may be left vertical or nearly so. Thus, at the section shown in Fig. 16, a rod reading is taken at the point *a*, the cut is determined at this point, and then the points *b*, *c*, *d*, *e*, *f*, and *g*

are located by stakes or marked where stakes cannot be driven. For each of the points *b, c, d, e, f,* and *g,* an elevation and a distance from the center are obtained. The elevations of points along the surface between slope stakes, including the center *h,* are taken before the excavation is started.

The spaces *bd* and *ce,* called *berms,* are left between the tops of the cut in the rock and the beginnings of the cut in the earth.

50. Computations for Compound Section.—The area of the rock in a compound section is computed by the methods already explained for sections in which the material is entirely of one class. The area of the earth is determined by dividing the figure into various triangles and trapezoids, calculating the area

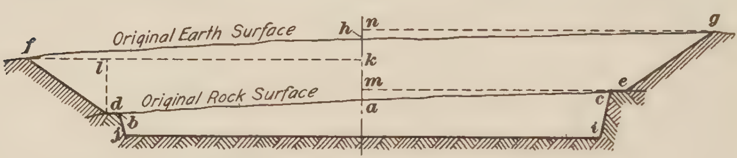


Fig. 16

of each part, and taking the sum of these areas. In the computations to determine the area of the earth, the elevations of the rock at points *b* and *c,* Fig. 16, may be ignored. A carefully drawn sketch on which are marked the depths and horizontal distances is very helpful in making the computations.

Example.—Compute the volume of earth and the volume of rock in the cut between Stas. 362 and 363 from the engineer's notes given below. The roadbed is 36 feet wide, the slope ratios are ¼ horizontal to 1 vertical for rock and 1½ to 1 for earth, and each berm is 1 foot wide.

Sta.	Earth			Rock				
	Left	Center	Right	Left		Center	Right	
363	$\frac{\text{C } 6.2}{26.2}$	C 7.2	$\frac{\text{C } 7.6}{26.0}$	$\frac{\text{C } 1.7}{19.5}$	$\frac{\text{C } 1.8}{18.5}$	C 3.2	$\frac{\text{C } 3.4}{18.9}$	$\frac{\text{C } 3.5}{19.9}$
362	$\frac{\text{C } 5.2}{24.6}$	C 6.4	$\frac{\text{C } 6.8}{25.7}$	$\frac{\text{C } 1.0}{19.3}$	$\frac{\text{C } 1.2}{18.3}$	C 2.6	$\frac{\text{C } 2.8}{18.7}$	$\frac{\text{C } 2.8}{19.7}$
	f	*h*	*g*	*d*	*b*	*a*	*c*	*e*

The letters in the last line have been added to indicate the corresponding points in Fig. 16, which may be considered to represent both cross-sections.

Solution.—*Volume of Rock.*—The section *bjic* at each station is a three-level section and formula 2, Art. 36, may be used to compute the area of the rock.

For Sta. 362, $a=\frac{b}{2s}=\frac{36}{2\times\frac{1}{4}}=72$ ft., $d=2.6$ ft., $w=18.3+18.7=37.0$ ft., and $b=36$ ft. Hence,

$$A_1=\tfrac{1}{2}(a+d)w-\tfrac{1}{2}ab=\tfrac{1}{2}\times(72+2.6)\times37-\tfrac{1}{2}\times72\times36=84.1 \text{ sq. ft.}$$

For Sta. 363, $a=72$ ft., as before, $d=3.2$ ft., $w=18.5+18.9=37.4$ ft., and $b=36$ ft. Hence,

$$A_2=\tfrac{1}{2}\times(72+3.2)\times37.4-\tfrac{1}{2}\times72\times36=110.2 \text{ sq. ft.}$$

From the formula of Art. 30,

$$V_1=\frac{l}{54}(A_1+A_2)=\tfrac{100}{54}\times(84.1+110.2)=360 \text{ cu. yd.}$$

The prismoidal correction, from the formula of Art. 37, is

$$C=\frac{l}{324}(w-w')(d'-d)$$
$$=\tfrac{100}{324}\times(37.0-37.4)(3.2-2.6)=-.07 \text{ cu. yd.}$$

which may be ignored.

Therefore, the corrected amount of excavation of rock between Stas. 362 and 363 is 360 cu. yd. Ans.

Volume of Earth.—The area *dfhgea*=area *fhk*+area *fld*+area *adlk* +area *aem*+area *megn*−area *hgn*. For Sta. 362,

area *fhk*	$=\frac{1}{2}fk\times hk$	$=\frac{1}{2}\times24.6\times(6.4-5.2)$	= 14.8 sq. ft.
area *fld*	$=\frac{1}{2}fl\times dl$	$=\frac{1}{2}\times(24.6-19.3)\times(5.2-1.0)$	= 11.1 sq. ft.
area *adlk*	$=\frac{1}{2}kl(dl+ak)$	$=\frac{1}{2}\times19.3\times[(5.2-1.0)+(5.2-2.6)]$	= 65.6 sq. ft.
area *aem*	$=\frac{1}{2}em\times am$	$=\frac{1}{2}\times19.7\times(2.8-2.6)$	= 2.0 sq. ft.
area *megn*	$=\frac{1}{2}(em+gn)mn$	$=\frac{1}{2}\times(19.7+25.7)\times(6.8-2.8)$	= 90.8 sq. ft.
−area *hgn*	$=-\frac{1}{2}gn\times hn$	$=-\frac{1}{2}\times25.7\times(6.8-6.4)$	=−5.1 sq. ft.
The total area of the earth at Sta. 362			=179.2 sq. ft.

In a similar way for Sta. 363,

area *fhk*	$=\frac{1}{2}\times26.2\times(7.2-6.2)$	= 13.1 sq. ft.
area *fld*	$=\frac{1}{2}\times(26.2-19.5)\times(6.2-1.7)$	= 15.1 sq. ft.
area *adlk*	$=\frac{1}{2}\times19.5\times[(6.2-1.7)+(6.2-3.2)]$	= 73.1 sq. ft.
area *aem*	$=\frac{1}{2}\times19.9\times(3.5-3.2)$	= 3.0 sq. ft.
area *megn*	$=\frac{1}{2}\times(19.9+26.0)\times(7.6-3.5)$	= 94.1 sq. ft.
−area *hgn*	$=-\frac{1}{2}\times26.0\times(7.6-7.2)$	=−5.2 sq. ft.
The total area of the earth at Sta. 363		=193.2 sq. ft.

The approximate volume of earth is

$$V_1=\frac{l}{54}(A_1+A_2)=\tfrac{100}{54}\times(179.2+193.2)=690 \text{ cu. yd.}$$

The approximate prismoidal correction can be found by the formula of Art. 37. For $l=100$ ft., $w=24.6+25.7=50.3$ ft., $w'=26.2+26.0=52.2$ ft., $d'=7.2-3.2=4.0$ ft. and $d=6.4-2.6=3.8$ ft.,

$$C=\frac{l}{324}(w-w')(d'-d)=\tfrac{100}{324}\times(50.3-52.2)\times(4.0-3.8)=-.12 \text{ cu. yd.,}$$

which may be ignored.

Therefore, the corrected amount of excavation of earth between Stas. 362 and 363 is 690 cu. yd. Ans.

51. Borrow Pits.—The term borrow pit is applied to an excavation made solely for the purpose of obtaining material with which to make a fill. Borrow pits are opened when there is a shortage of suitable material from adjacent cuts. Sometimes, a borrow pit is made by widening a cut, as illustrated in Fig. 17, with the idea that the added width, properly graded beside the roadbed, may ultimately prove of use as a place for side tracks. In any case, since payment for earthwork is invariably made on the basis of the amount of earth excavated, the excavation is made on regular lines, so that it may be readily measured. Cross-sections of such borrow pits should be taken

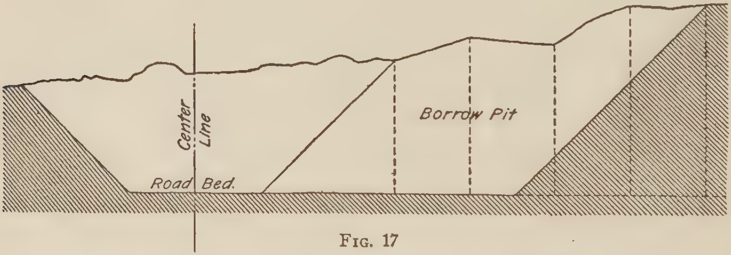


FIG. 17

at regular intervals, and the amount of material excavated should be computed by the method of average end areas, with the application of the prismoidal correction when necessary.

In Fig. 18 is shown a perspective view of a cut with a borrow pit adjoining it, the borrow pit being made by widening the cut. The cross-sections to be taken are indicated by the dotted lines. It should be noted that at one end the terminal solid is the

figure *abcdef*, which may be assumed to be a wedge. At the other end the borrow pit runs to the natural end of the cut, and the terminal solid at that end is the pyramid *mnpq*. The volumes of these terminal solids are best obtained by the application of the rules of geometry. Thus the volume of a pyramid is equal to one-third of the product of the area of the base and

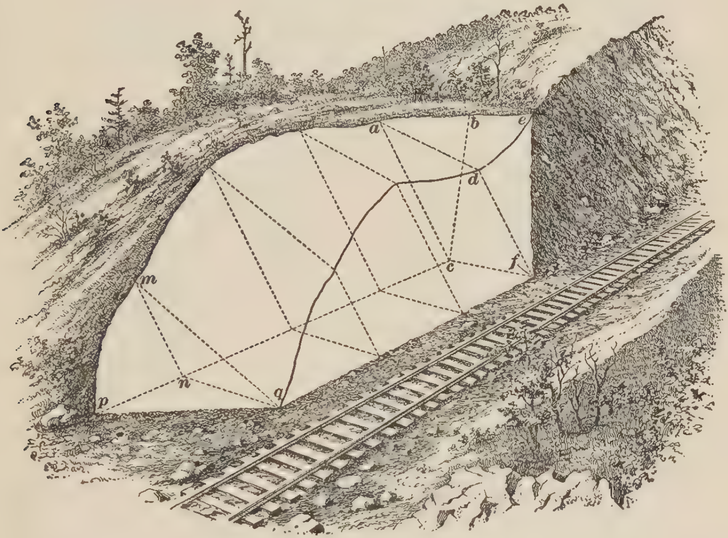


FIG. 18

the altitude, and the volume of a wedge is equal to one-half of the product of the area of the base and the altitude. The area of each base is computed from the notes of the cross-section which forms the base. The altitude of the solid is the difference between the station of the end point or line and the station of the base.

52. Measurement of Excavation in Irregular Borrow Pits. Often the ground from which earth is to be borrowed, is divided into rectangles from 10 to 30 feet on a side. For rough ground or where great accuracy is required, the shorter distances are used. At each corner of a rectangular division, elevations are taken before and after excavation; then the difference between

the original and final elevations at the corner gives the cut at that point. The average of the cuts in feet at the corners of any division, multiplied by the area of the division in square feet, gives the volume, in cubic feet, of the excavation in the given division. This volume may be reduced to cubic yards by dividing by 27.

EXAMPLE.—The elevations taken before excavation at the four corners of a 20-foot square borrow pit are 86.3, 85.4, 89.1, and 87.6 feet; after excavation, the elevations of the same four corners are, respectively, 76.1, 78.7, 80.2, and 79.6 feet.

What is the number of cubic yards excavated?

SOLUTION.—The cuts are as follows:

At the first corner,	86.3−76.1=10.2 ft.
at the second corner,	85.4−78.7= 6.7 ft.
at the third corner,	89.1−80.2= 8.9 ft.
at the fourth corner,	87.6−79.6= 8.0 ft.
The total cut	=33.8 ft.
The average cut is $\frac{33.8}{4}$	= 8.45 ft.

The volume of excavation from the pit is

$20\times20\times8.45=3{,}380$ cu. ft. or 125 cu. yd. Ans.

53. Allowance for Ditches.—In the computations of volumes in the preceding articles, no allowance was made for ditches, such as are shown in Figs. 1 and 2. As explained in Art. 15, there are two ditches, aa' and cc' in Fig. 1, and often a third ditch such as that near b, for which allowance should be made in cuts. An allowance is sometimes also made in the case of fills for the required ditch such as that near e in Fig. 2. The area of each section in cuts may be increased by the area of the ditches, and the volumes computed by the formula of Art. 30, but, since each ditch is a prism, it is generally more convenient to compute the volume of the ditches separately.

If A is the area, in square feet, of the cross-section of a ditch, and l is the length of the ditch, in feet, the volume V_d, in cubic yards, is given by the formula

$$V_d=\frac{Al}{27}$$

The material excavated from the ditches is usually available for embankment, but that excavated from the ditch near *b*, Fig. 1, is often piled on the ground between the ditch and the point *b*.

Example.—If two ditches in a cut are each 4 feet wide at the top, 1 foot wide at the bottom, and 1 foot deep, what is the additional excavation required for each 100 feet of cut on account of these ditches?

Solution.—The cross-section of each ditch is a trapezoid whose bases are 4 ft. and 1 ft., and whose altitude is 1 ft. Therefore,

$$A=\tfrac{1}{2}\times(4+1)\times 1=2.5 \text{ sq. ft. and } V_d=\frac{2.5\times 100}{27}=9.3 \text{ cu. yd.}$$

Since there are two ditches, the additional excavation between two stations 100 ft. apart in cuts is

$$2\times 9.3=19 \text{ cu. yd.} \quad \text{Ans.}$$

54. Additions and Subtractions for Highway Sections.—It is usual in sections for highways, as in those for railroads, to obtain the area to some convenient line and then add or subtract the amount necessary to obtain the correct area. For example,

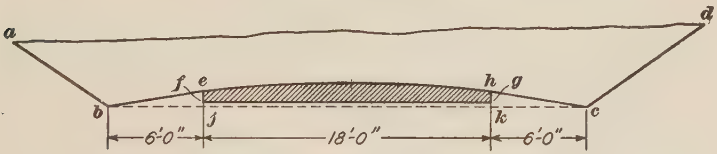


Fig. 19

in computations for the section shown in Fig. 19, where the excavation is to be made to the line *abefghcd*, a cut would first be figured down to the line *abcd* and then the area *befghc* would be subtracted.

Example.—In Fig. 19, *bc* is 30 feet, and the side-slope ratios are $1\frac{1}{2}$ horizontal to 1 vertical. The width *fg* is 18 feet, *ef* is 6 inches, and *fg* is 3 inches above *jk*. What is the area of *befghc*?

Solution.—The triangle *bej* has a base of 6 ft. and an altitude of 6 in. +3 in.=9 in. Hence, its area is $\frac{1}{2}\times 6\times\frac{9}{12}=2.25$ sq. ft. The area of *chk* is also 2.25 sq. ft., and the area of *fgkj* is $18\times\frac{3}{12}=4.5$ sq. ft. Therefore, the total area to be subtracted is $2.25+2.25+4.5=9$ sq. ft. Ans.

CORRECTION FOR CURVATURE

GENERAL CASE

55. Introductory Explanations.—The previous calculations of the volume of earthwork between two sections have been made on the assumption that the center line of the roadbed is straight and that the cross-sections are, therefore, parallel to each other. However, a considerable portion of a railroad or highway has a curved center line and the cross-sections in these portions are radial and not parallel to each other.

In Fig. 20 is shown a portion of an embankment on a curve; r_1r_2 is the curved center line of the roadbed and O is the center of the curve. For a curved railroad or highway, the curved

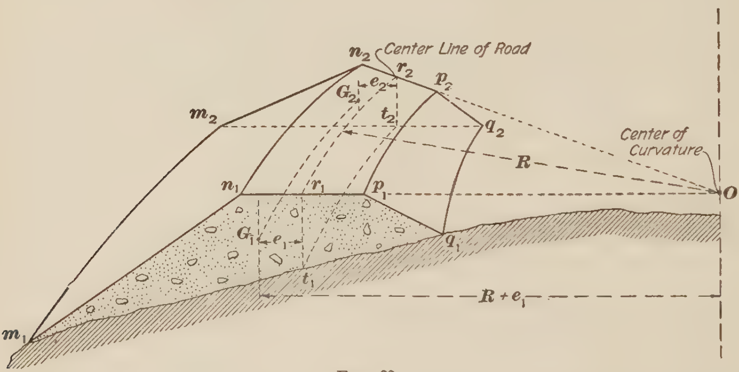


FIG. 20

distances between corresponding points on adjacent cross-sections vary: on the outside of the curve, these distances are greater than the center line lengths; on the inside of the curve, they are less. The distance which should be employed in computing the volume of a curved embankment or cut is the curved length between the centers of gravity of the two adjacent cross-sections. This distance multiplied by the average end area is equal to the approximate volume. In practice it has been found more convenient to compute the volume as though the center line were straight and then add or subtract the proper correction for curvature.

56. Eccentricity of Cross-Section.—If the point G_1 in Fig. 20 is the center of gravity of the section $m_1n_1p_1q_1$, then the horizontal distance e_1 from the center line r_1t_1 to the point G_1 represents the eccentricity of the section. Similarly, if the point G_2 is the center of gravity of the section $m_2n_2p_2q_2$, the eccentricity of that section is the distance e_2 from the center line r_2t_2 to the point G_2. The eccentricity of a cross-section may be determined as follows:

The section is first divided into a number of parts, usually triangles, for which the centers of gravity and the areas can be easily found, as in Fig. 21. The area of each part is multiplied by the distance from its center of gravity to the center line of the roadbed; distances measured from the center line of the road-

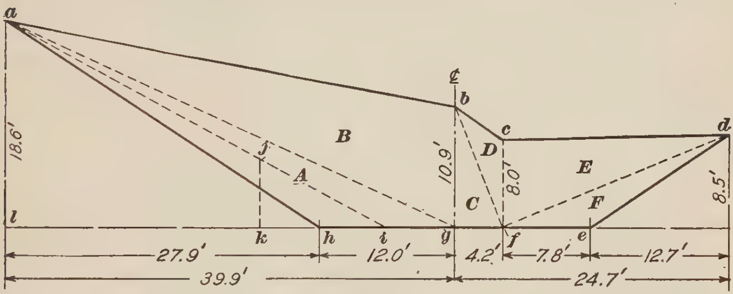


FIG. 21

bed away from the center of the curve are considered positive, and distances toward such center are considered negative. The products thus obtained are added algebraically, and the required eccentricity is found by dividing the sum of these products by the area of the entire section. A positive value of the eccentricity indicates that the center of gravity of the section is outside the center line of the roadbed, and a negative value indicates that the center of gravity is inside the center line.

A comparatively simple way of computing the horizontal distance from the center line of the roadbed to the center of gravity of any triangular part of the cross-section is by finding one-third of the sum of the distances from the center line to the three vertexes of the triangle. In the case of triangle A of Fig. 21, the horizontal distance from the center line bg of the roadbed to each

vertex of the triangle is zero to vertex g, gh to vertex h, and gl to vertex a; hence, the distance gk to the center of gravity j of the triangle is $\frac{1}{3}(0+gh+gl)$. If one side of the triangle is vertical, as in the case of triangle B, C, D, or E, the vertical side may be treated as the base, and the horizontal distance from that side to the center of gravity of the triangle is one-third of the altitude. Thus, in triangle D, the distance from the vertical side cf to the center of gravity is $\frac{1}{3}fg$, and the distance from the center line bg to the center of gravity is $\frac{2}{3}fg$.

Example.—Locate the center of gravity of the cross-section at Sta. 128 from the notes in Table IV in Art. 41, assuming that the left-hand side is on the inside of the center line.

Solution.—The first step is to prepare a sketch as shown in Fig. 21, in which the section whose center of gravity is to be located is represented by the figure *abcdefgh*. The figure is divided by dotted lines into six triangles, A, B, C, D, E, and F. Lever arms to the right of the center line are considered as positive; those to the left, as negative. The computations may be tabulated as follows:

Triangle	Area, in Square Feet	Arm, in Feet	Product
A	$\frac{1}{2}\times12.0\times18.6=111.6$	$-\frac{1}{3}(0+12+39.9)=-17.3$	−1,931
B	$\frac{1}{2}\times10.9\times39.9=217.5$	$-\frac{1}{3}\times39.9=-13.3$	−2,893
C	$\frac{1}{2}\times10.9\times4.2=22.9$	$\frac{1}{3}\times4.2=+1.4$	+ 32
D	$\frac{1}{2}\times8.0\times4.2=16.8$	$\frac{2}{3}\times4.2=+2.8$	+ 47
E	$\frac{1}{2}\times8.0\times20.5=82.0$	$4.2+\frac{1}{3}\times20.5=+11.0$	+ 902
F	$\frac{1}{2}\times7.8\times8.5=33.2$	$\frac{1}{3}\times(4.2+12+24.7)=+13.6$	+ 452
Total	484.0		−3,391

The distance of the center of gravity from the center line is $-3{,}391\div484=-7.0$ ft. Since the answer is negative, the center of gravity is to the left, or inside, of the center line. Ans.

57. Curvature Correction.—When the center of gravity in each of the sections of a curved embankment or cut of uniform cross-section is at a distance e from a center line of radius R, then the amount by which the curved distance between the centers of gravity of any two sections differs from the distance measured along the center line is to the latter distance in the ratio $e:R$. Since the volume for a straight roadbed is the product of the average area A and the center-line length l, or Al, the correction to be applied on account of the curvature is

$\frac{e}{R} \times Al = \frac{l}{R} Ae$. As in Fig. 20, the values of Ae generally differ for the two sections used in computing the volume, and the average of the two products obtained when each area is multiplied by the corresponding eccentricity, or $\frac{1}{2}(A_1e_1 + A_2e_2)$, should then be used in place of Ae. The correction for curvature is usually expressed in cubic yards, and it can be found by the formula

$$C_c = \frac{l}{54R}(A_1e_1 + A_2e_2)$$

in which C_c = correction to be applied on account of curvature, in cubic yards;
l = length between sections, measured along curved center line, in feet;
R = radius of center line, in feet;
A_1 = area of first section, in square feet;
e_1 = eccentricity of first section, in feet;
A_2 = area of second section, in square feet;
e_2 = eccentricity of second section, in feet.

In this formula, e_1 or e_2 should be considered positive if on the outside of the center line, and negative if on the inside. The correction C_c is added to the volume of earthwork for a straight roadbed if its value is positive, and is subtracted if its value is negative. When the degree of curvature is given, the radius of the curve can be obtained with sufficient accuracy by dividing 5,730 by the degree of curvature. The radius need be computed only to the nearest foot.

EXAMPLE.—Find the correction for curvature between Stas. 127 and 128 from the notes in Table IV, Art. 41, if the roadbed is on a curve to the left whose radius is 1,146 feet.

SOLUTION.—The center of gravity of the area of the cut at Sta. 128, as located in the preceding article, is 7.0 ft. inside the center line; hence, e_2 is -7 ft. In a similar way, the center of gravity of the area of the cut at Sta. 127 may be found to be 1.3 ft. inside he center line and $e_1 = -1.3$ ft. From Table IV, A_1, the area at Sta. 127, is 439.4 sq. ft. and A_2, the area at Sta. 128, is 483.9 sq. ft. Then, by the formula of this article,

$$C_c = \frac{1}{54R}(A_1e_1 + A_2e_2) = \frac{100}{54 \times 1{,}146} \times [439.4 \times (-1.3) + 483.9 \times (-7.0)]$$
$$= -6 \text{ cu. yd.} \quad \text{Ans.}$$

The minus sign indicates that the correction is to be subtracted from the volume as found for a straight roadbed.

58. Final Adjusted Volume.—The first step in determining the volume of earthwork for a length of curved track or highway is to compute the volume by averaging end areas just as if the track were straight. Then the prismoidal correction is made, if considered desirable by the engineer. Finally, the correction for curvature is applied. The result is the desired volume, called the *adjusted volume*. Thus, in the example of the preceding article, the corrected volume for a straight roadbed is, from Table IV, 1,710 cubic yards, the correction for curvature is −6 cubic yards, and the adjusted volume is 1,710 −6 = 1,704 cubic yards.

CURVATURE CORRECTION FOR THREE-LEVEL SECTIONS

59. Calculation of Correction.—For a large amount of earthwork, three-level sections give sufficiently accurate results. The corrections for curvature for such cases are computed by the general formula of Art. 57. However, in determining the product of the area A and the eccentricity e for a typical three-level section, such as is shown in Fig. 9, the computations are simplified by the addition of the grade triangle *mnt*. Since the eccentricity of that triangle is zero, the product of its area and eccentricity is also zero, and the product Ae for the three-level section *mpqt* about the center line is equal to the product Ae for the area *nqcp* about the same axis. However, by including the grade triangle, the area *nqcp* can be conveniently divided into the two triangles *cnq* and *cnp*. For the triangle *cnq*,

$$A = \tfrac{1}{2}(a+d)w_l$$
$$e = \tfrac{1}{3}w_l$$
$$Ae = \tfrac{1}{2}(a+d) \times \tfrac{1}{3}w_l^2$$

For the triangle *cnp*,

$$A = \tfrac{1}{2}(a+d)w_r$$
$$e = \tfrac{1}{3}w_r$$
$$Ae = \tfrac{1}{2}(a+d) \times \tfrac{1}{3}w_r^2$$

If the curve is to the right, the area *cnq* is on the outside of the center line of the curve and the curvature correction for that area is positive; the area *cnp* is on the inside of the center line of the curve and the curvature correction for that area is negative. The algebraic sum of the values of Ae for the triangles *cnq* and *cnp* is

$$Ae=\tfrac{1}{2}(a+d)\times\tfrac{1}{3}w_l^2-\tfrac{1}{2}(a+d)\times\tfrac{1}{3}w_r^2=\tfrac{1}{2}(a+d)\times\tfrac{1}{3}(w_l^2-w_r^2)$$
$$=\tfrac{1}{2}(a+d)\times\tfrac{1}{3}(w_l+w_r)(w_l-w_r)$$

Hence when the curve is to the right, the value of Ae for the three-level section is expressed by the formula

$$Ae=\tfrac{1}{2}(a+d)w\times\tfrac{1}{3}(w_l-w_r) \qquad (1)$$

In a similar way, if the curve is to the left, the curvature correction for the area *cnq* is negative and that for the area *cnp* is positive and the sum of the values of Ae is

$$Ae=\tfrac{1}{2}(a+d)\times\tfrac{1}{3}w_r^2-\tfrac{1}{2}(a+d)\times\tfrac{1}{3}w_l^2$$

Hence when the curve is to the left,

$$Ae=\tfrac{1}{2}(a+d)w\times\tfrac{1}{3}(w_r-w_l) \qquad (2)$$

60. Illustrative Example.—In practice, the use of the formulas of the preceding article is often made easier by the fact that $\frac{1}{2}(a+d)w$ has been already computed as for a straight roadbed, as in the notes for three-level sections shown in Table III, Art 38.

Example.—Compute the adjusted volume of the excavation between Stas. 22 and 23 in Table III, if these stations are on a 7° curve to the right.

Solution.—Here $l=100$ ft. and $R=\frac{5{,}730}{7}=819$ ft. The values given in Table III for Sta. 22 are: from column 2, $w_l=16.1$ ft.; from column 4, $w_r=30.2$ ft.; and from column 7, $\frac{1}{2}(a+d)w=312.5$. Hence, from formula 1 of the preceding article,

$$A_1e_1=\tfrac{1}{2}(a+d)w\times\tfrac{1}{3}(w_l-w_r)$$
$$=312.5\times\tfrac{1}{3}\times(16.1-30.2)=-1{,}469$$

For Sta. 23, $\frac{1}{2}(a+d)w=414.2$; $w_l=18.2$ ft.; and $w_r=31.4$ ft. Therefore,

$$A_2e_2=414.2\times\tfrac{1}{3}\times(18.2-31.4)=-1{,}822$$

The curvature correction between Stas. 22 and 23 is, by the formula of Art. 57,

$$C_c=\frac{l}{54R}(A_1e_1+A_2e_2)=\frac{100}{54\times819}\times(-1{,}469-1{,}822)=-7 \text{ cu yd.}$$

The corrected volume for a straight roadbed is given as 1,044 cu. yd. in the last column of Table III; hence the adjusted volume for the roadbed when curved 7° to the right is 1,044−7=1,037 cu. yd. Ans.

EXAMPLES FOR PRACTICE

Solve the following examples, assuming that the excavation for which the notes and calculations are given in Table III is on a 7° curve to the right.

1. Find the adjusted volume of excavation between Sta. 23 and Sta. 24. Ans. 1,578 cu. yd.

2. Find the adjusted volume of excavation between Sta. 24 and Sta. 24+35. Ans. 522 cu. yd.

3. Find the adjusted volume of excavation between Sta. 24+35 and Sta. 25. Ans. 402 cu. yd.

CORRECTION FOR CURVATURE IN SIDE-HILL WORK

61. Introduction.—Where the ground is steep at right angles to a curved center line the curvature correction is comparatively large and should always be applied. Furthermore, where there is both cut and fill at the same section on a curve, the curvature correction must be applied separately to each. In side-hill work, a situation of this sort is particularly likely to arise, as shown in Fig. 22. The original surface is represented by the irregular line knk'; the graded surface by the lines km, mm', and $m'k'$; the cut, by kmn; and the fill, by $k'm'n$.

62. Calculation of Correction.—The correction for curvature in either cut or fill in side-hill work is computed by applying the formula of Art. 57. An exact method of determining the area and eccentricity of an irregular shape, which may always be used, has been explained in Art. 56. Very often, however, the product of the area and eccentricity in side-hill work differs but little from that which would be obtained if the given section was assumed to be triangular. For example, in Fig. 22, the product of the area and the eccentricity for the irregular figure kmn is about the same as the product of the area and the eccentricity for the triangle kmn. The eccentricity of a triangular area may be conveniently found by the application of the principle that the distance from the center line of the road-

bed to the center of gravity of the triangle is equal to one-third the algebraic sum of the distances from that line to the vertexes of the triangle.

This principle may be applied to a given triangular section in side-hill work in the following way: Add together the length of the half-base and the distance from the center line to the slope stake. If the point at grade is on the same side of the center line as the slope stake, add also its distance from the center line; if it is on the opposite side, subtract its distance from the center line. Then one-third of the result is the eccentricity, which is on the same side of the center line as the slope stake.

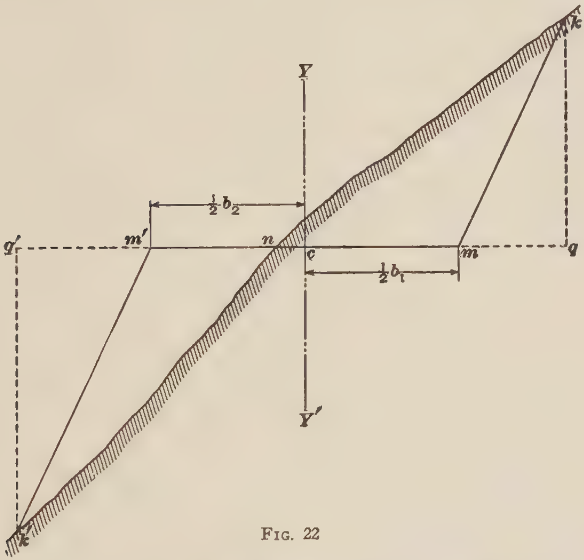


FIG. 22

Thus, in Fig. 22, the eccentricity of the triangle *kmn* is $\dfrac{cm+cq-cn}{3}$, and the eccentricity of the triangle *k′m′n* is $\dfrac{cm'+cq'+cn}{3}$.

EXAMPLE 1.—Compute the curvature correction for the cut between Stas. 32 and 33 from the following notes. The roadbed is 24 feet wide

in cuts and 16 feet wide in fills; the slope ratios are 1½ to 1; and both stations are on a 9° curve to the right.

Station	Left		Center	Right	
33	$\frac{\text{F } 18.8}{36.2}$		F 1.3	$\frac{0}{3.4}$	$\frac{\text{C } 17.1}{37.7}$
32	$\frac{\text{F } 15.0}{30.5}$	$\frac{0}{4.4}$	C 2.0		$\frac{\text{C } 14.9}{34.4}$

Solution.—The section at Sta. 32 is represented by Fig. 22, and that at Sta. 33 by Fig. 23.

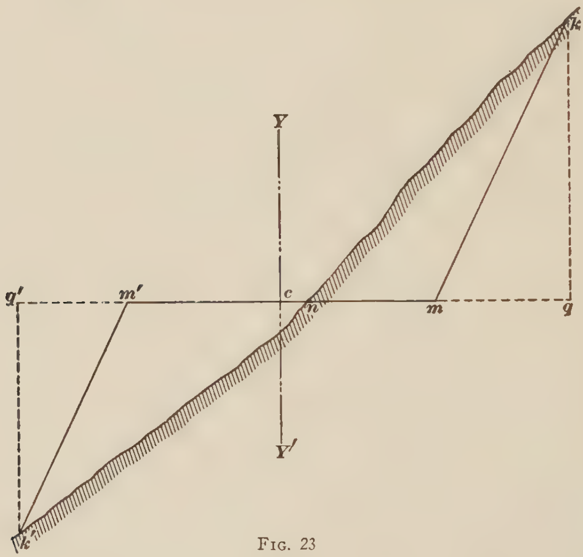


Fig. 23

In the formula of Art. 57, or $C_c = \frac{l}{54R}(A_1e_1 + A_2e_2)$, the distance l between Sta. 32 and Sta. 33 is 100 ft., and $R = \frac{5{,}730}{9} = 637$ ft.

For the purpose of computing the curvature correction, the sections in both cut and fill may in this case be considered as triangles.

For the cut at Sta. 32,

$$A_1 = \text{area of triangle } nmk = \tfrac{1}{2}nm \times kq$$
$$= \tfrac{1}{2} \times (4.4 + \tfrac{24}{2}) \times 14.9 = 122.2 \text{ sq. ft.}$$

$$e_1=\frac{12+34.4-4.4}{3}=14.0 \text{ ft.}$$

For the cut at Sta. 33, Fig. 23,

$$A_2=\text{area of triangle } nmk=\tfrac{1}{2}nm\times kq$$
$$=\tfrac{1}{2}\times(\tfrac{24}{2}-3.4)\times 17.1=73.5 \text{ sq. ft.}$$
$$e_2=\frac{12+37.7+3.4}{3}=17.7 \text{ ft.}$$

Since e_1 and e_2 are both on the inside of the center line of the curve, they are negative. Hence,

$$C_c=\frac{100}{54\times 637}\times[122.2\times(-14.0)+73.5\times(-17.7)]=-9 \text{ cu. yd.} \quad \text{Ans.}$$

In other words, 9 cu. yd. is to be subtracted from the volume obtained by considering the track as straight.

Example 2.—Compute the curvature correction for the fill between Stas. 32 and 33 for conditions stated in the preceding example.

Solution.—As in example 1, $l=100$ ft. and $R=637$ ft. For the fill at Sta. 32,

$$A_1=\textit{area of triangle } nm'k' \text{ in Fig. } 22=\tfrac{1}{2}nm'\times k'q'$$
$$=\tfrac{1}{2}\times(\tfrac{16}{2}-4.4)\times 15.0=27.0 \text{ sq. ft.}$$
$$e_1=\frac{8+30.5+4.4}{3}=14.3 \text{ ft.}$$

For the fill at Sta. 33,

$$A_2=\text{area of triangle } nm'k' \text{ in Fig. } 23=\tfrac{1}{2}nm'\times k'q'$$
$$=\tfrac{1}{2}\times(\tfrac{16}{2}+3.4)\times 18.8=107.2 \text{ sq. ft.}$$
$$e_2=\frac{8+36.2-3.4}{3}=13.6 \text{ ft.}$$

In this case, e_1 and e_2 are both positive, since they are on the outside of the center line of the curve, and

$$C_c=\frac{100}{54\times 637}\times(27.0\times 14.3+107.2\times 13.6)=5 \text{ cu. yd.} \quad \text{Ans.}$$

Hence, 5 cu. yd. is to be added to the volume obtained by considering the track as straight.

EXAMPLES FOR PRACTICE

1. The following notes apply to a 10° curve to the left. The roadbed is 24 feet wide in the cuts, and 16 feet wide in the fills. Find the correction for curvature for the cut between Stas. 161 and 162, if the sections in cut and fill are assumed to be triangular.

Station	Left		Center	Right	
162	$\frac{\text{F } 25.5}{46.3}$	$\frac{0}{1.5}$	C 1.2		$\frac{\text{C } 22.4}{45.6}$
161	$\frac{\text{F } 28.5}{50.8}$		F 2.4	$\frac{0}{6.3}$	$\frac{\text{C } 12.2}{30.3}$

Ans. $C_c = +11$ cu. yd.

2. Find the correction for curvature for the fill in the preceding example. Ans. $C_c = -17$ cu. yd.

3. If the roadbed to which the following notes refer is 20 feet wide in the cuts, and is on a 10° curve to the right, find the correction for curvature for the cut between Stas. 22+40 and 23. The sections in cut and fill may be considered as triangular.

Station	Left		Center	Right	
23	$\frac{\text{F } 18.9}{36.4}$	$\frac{0}{6.0}$	C 2 3		$\frac{\text{C } 20.0}{40.0}$
22+40	$\frac{\text{F } 18.0}{35.0}$		F 0.8	$\frac{0}{2.0}$	$\frac{\text{C } 18.0}{37.0}$

Ans. $C_c = -7$ cu. yd.

DISPOSAL OF CUT AND FILL

FORMATION OF EMBANKMENTS

SHRINKAGE AND SWELL

63. Shrinkage.—It is usually observed that when earth is excavated, its volume is at first greater than that of the space it originally occupied; but, after it has been placed in an embankment, the earth contracts or shrinks so that its volume is less than in the original position. In general, the reasons for this shrinkage are: (1) the earth in its natural state is rendered more or less porous by the continued action of frost; (2) roots of trees and plants penetrate into the soil and subsequently decay, leaving cavities or pores; (3) the soil is made porous by the soluble action of percolating water; and (4) some soil is

lost when it is moved or transported from the excavation to the embankment. The amount of the reduction of the volume is also dependent on the method of forming the embankment, weather conditions during construction, etc. On an average, shrinkage generally amounts to about 10 per cent., although in the case of a very loose vegetable soil, it may be as much as 25 per cent. The approximate per cent. of shrinkage for the various kinds of soils is as follows: sand and gravel, 8; clay, 10; loam, 12; and loose vegetable soil, 15. For example, it may be expected that 1,000 cubic yards of excavated gravel will make $1,000-.08 \times 1,000$ or 920 cubic yards of fill. Also, since 1 cubic yard of cut in gravel makes only .92 cubic yard of fill, 1,000 cubic yards of fill will require $\frac{1,000}{.92}$ or 1,087 cubic yards of cut.

EXAMPLE 1.—The volume of a cut through clay is 1,630 cubic yards. How many cubic yards is contained in an embankment made from this material?

SOLUTION.—Since clay shrinks 10%, 1 cu. yd. of clay in cut forms .9 cu. yd. in fill and the required volume is $.9 \times 1,630 = 1,467$ or about 1,470 cu. yd. Ans.

EXAMPLE 2.—How many cubic yards of excavation in gravel are required to form an embankment of 2,200 cubic yards?

SOLUTION.—Since 1 cu. yd. of cut in gravel makes .92 cu. yd. in fill, the amount of cut required for a fill of 2,200 cu. yd. is $\frac{2,200}{.92} = 2,391$, or about 2,390 cu. yd. Ans.

64. Swell of Rock.—When an embankment is made of broken-up hard rock, it has a volume from 40 to 80 per cent. larger than its original volume in the cut, and there is practically no subsequent settling of the embankment. This increase in volume is called the swell of rock. It varies a great deal with conditions, but 50 per cent. may be taken as the average swell.

EXAMPLE.—How many cubic yards of embankment may be expected from a rock cut containing 4,500 cubic yards?

SOLUTION.—Since the swell may be expected to be 50%, the amount of embankment is $4,500 + .5 \times 4,500 = 6,750$ cu. yd. Ans.

65. Uncertainties in Estimating Shrinkage.—There is a great deal of uncertainty about the estimates for shrinkage and swell, particularly if the material is intermediate between earth and rock. Furthermore, in some locations the top soil is soft, loamy earth and the material below it becomes gradually harder until firm rock is reached. The shrinkage for the top earth will be about 10 per cent. and the swell for the hard rock, about 50 per cent.; the allowance for the intermediate material must be determined by the judgment of the engineer.

Sometimes rock that is hard when excavated soon crumbles into dirt on exposure to the air. Such rock will swell immediately after excavation. Gradually, however, it will shrink to about the volume occupied by an equal amount of earth.

On account of the uncertainties in shrinkage and swell, it is the custom to base the computations for all earthwork and rockwork on the volume of the material in its original position, whether it is in a cut or in a borrow pit.

EXAMPLES FOR PRACTICE

1. Find the amount of embankment that can be formed from a cut of 3,200 cubic yards in loam. Ans. 2,820 cu. yd.

2. Find the amount of embankment that can be formed from a cut of 3,000 cubic yards in hard rock. Ans. 4,500 cu. yd.

3. Find the volume required for a borrow pit in sandy soil to furnish 3,590 cubic yards of embankment. Ans. 3,900 cu. yd.

SUBSIDENCE

66. Amount of Subsidence.—The weight of an embankment and the vibration due to heavy traffic compress the natural soil on which the embankment is placed. This action, which differs from the shrinkage of the material used in fill, is called subsidence. The amount of the subsidence depends on the depth of the embankment and the nature of the soil. The subsidence is negligible on a rocky soil, reaches an appreciable value on moderately firm soils, and becomes very large on soft or marshy ground. This means that when an embankment is formed on anything but a rocky soil, material must be used exceeding the volume called for by the nominal cross-section

above the original ground surface. In extreme cases more material is needed below the original surface than in the embankment above. The percentage of height of an embankment to be allowed for subsidence is greater for a low embankment than for a high one, because in the former the area of the base is less and the tamping action of the traffic is more direct and effective.

The extent of subsidence cannot be accurately foretold; even more difficulty is met than in predicting the ultimate shrinkage of a volume of excavated material that is formed into an embankment. Hence, the engineer must use judgment, aided perhaps by borings in soft soil, to estimate the subsidence. When subsidence is ignored, the usual result is a sag of the roadbed.

67. **Additional Height for Embankments.**—A fill should be left at an elevation above grade to allow for the expected settlement caused by shrinkage and by subsidence, for some time is required for these actions to take place. The weight of a partially finished embankment, and the carts and trucks passing over a fill, cause some shrinkage and some subsidence. Compacting the roadbed will hasten the shrinkage and tend to reduce future shrinkage. If heavy weights like rollers are used, future subsidence will also be lessened. As an embankment of hard rock will not shrink, only an allowance for subsidence need be made.

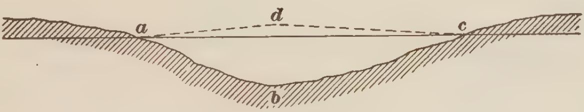


FIG. 24

68. **Allowance for Shrinkage and Subsidence.**—The method of allowing for settlement of the fill because of shrinkage and subsidence is shown in Fig. 24, in which *abc* represents the natural surface of the ground and *ac* is the adopted grade line. Directly over the lowest point *b*. the embankment is built up to the high point *d*, and is gradually sloped down to points

a and *c*, forming two grade lines *ad* and *dc*. After shrinkage and subsidence take place in the embankment *abc*, the roadbed reaches approximately the grade line *ac*. Whatever may be the percentage of shrinkage and subsidence, the settlement of an embankment due to these combined effects will be greater for a high than for a low embankment. The allowance usually made for settlement is from 5 to 15 per cent. of the height of the embankment, although it is claimed by many engineers that 2 or 3 per cent. is sufficient. In making this allowance, it is better to leave the embankment a little low, as it is easier to raise the finished surface than to lower it.

MOVING CUT TO FILL

METHODS OF FORMING EMBANKMENTS

69. Usual Methods.—There are various methods of excavating, transporting, and depositing earth for an embankment. If power equipment is not employed, it is necessary first to loosen the earth, unless it is sand or sandy loam. The loosening is usually done by means of picks or plows, and the earth is loaded with shovels into dump-wagons and transported to the fill. Carts for short hauls and wheelbarrows for very short hauls are sometimes employed in transporting earth from cut to fill. For embankments of low and moderate heights and for comparatively short hauls, the material may be excavated and moved to the place of deposit by means of scrapers. However, the employment of power shovels for loosening and loading earth is generally found most economical in all cases where the volume of earth is sufficient to justify the cost of installation, and such shovels are, therefore, widely used in highway and railroad work. They can excavate almost any material except solid rock, and they can even load rock that has been broken up into small pieces. For the usual fills in highway and railroad work, the earth is hauled to the place where it is to be deposited by team-drawn dump-wagons, motor trucks, large dump-wagons drawn by tractors, or dump-cars operating on light tracks. The last method is often used in railroad construction but very seldom in highway construction. For large

and long fills in railroad work a common method is to build a wooden trestle from which material is deposited by dumping from cars. When the fill reaches the top of the trestle, the deck of the trestle, with the ties and rails, is removed and shifted laterally. More filling is then deposited and the tracks are again shifted; these operations are repeated until the embankment reaches its proper width.

On side-hill work it is advisable to roughen the natural surface of the hillside by cutting steps or by plowing furrows, so that the deposited material will have a mechanical bond with the surface of the ground, thus guarding against the slipping of the embankment.

70. Construction of Temporary Trestle. — In railroad work, it is sometimes found economical to postpone the construction of an embankment by building a temporary wooden trestle to grade. The road can then be immediately opened to traffic and the embankment is constructed later as the material is procured from other places along the line. The material is hauled to the site and dumped from the trestle until a solid embankment of earth is formed under and around the trestle. This method of construction enables a company to start operating the road at a low first cost and to complete the embankment as the financial condition of the company warrants it. Furthermore, by this method, material along the line that otherwise would be wasted can be utilized to good advantage and large borrow pits in the immediate vicinity of the trestle are not needed.

HAULAGE

71. Limit of Free Haul and Overhaul.—The most variable factor in the cost of earthwork, and the one that in some cases is the largest single item, is that which depends on the length of haul, or the distance through which excavated material must be transported. Specifications usually require that a contractor shall deposit excavated material at any place designated by the engineer, and that his bid per cubic yard shall cover the cost of such haul, provided that the distance does not exceed some specified limit, say 500 or 1,000 feet. This extreme distance is

called the *limit of free haul*. It is also specified that, if the haul exceeds this distance, there shall be an extra allowance per cubic yard for each distance of 100 feet in excess of the free limit. The sum of all the products obtained by multiplying each cubic yard of earth measured in place of excavation by the number of stations that it is hauled beyond the specified limit of free haul is called the *overhaul*. For example, suppose that 150 cubic yards is hauled 743 feet on a job where the limit of free haul is 500 feet. Then the distance in excess of the free-haul limit is $743-500=243$ feet or 2.43 stations and the overhaul is 150×2.43 or 364.5 cubic yards hauled 100 feet.

72. Computation of Haulage.—The total haulage for a given portion of the line is the sum of all the products obtained by multiplying each volume by the distance through which that volume is hauled. This haulage equals the total number of cubic yards excavated multiplied by the distance between the

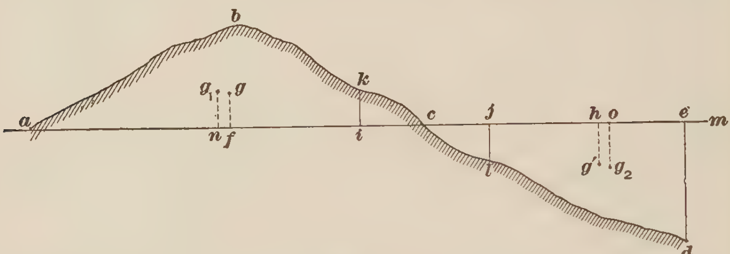


Fig. 25

center of gravity of the material in its original position and the center of gravity of the embankment formed by that same material. In Fig. 25 is shown a portion of the profile of a roadbed near a point in which it passes from a cut to a fill. If all the material of the cut *abc*, whose center of gravity is at g, is deposited in the position *cde*, with its center of gravity at g', the total haulage is the product of the volume of the cut *abc* and the distance *fh* between the centers of gravity g and g' of the cut and fill, respectively.

73. Compensation for Haulage.—If the distance *ae* in Fig. 25 is less than the limit of free haul, the contractor is not entitled to extra compensation for transporting the material

from the cut to the fill. Therefore, when all material from a cut is deposited within the specified limit of free haul, no computation of haul need be made. Furthermore, if there is overhaul for any cut, the contractor receives extra compensation only for that portion of the cut which is carried beyond the specified limit. Thus, if ij in Fig. 25 is a distance equal to the limit of free haul, and the material cik is deposited at cjl, no extra charge is admissible for this material. But, every cubic yard excavated from a point to the left of i and transported to an embankment on the right of j is overhauled, and extra compensation must be allowed for this.

74. Computing Overhaul.—The first step in computing the overhaul between a and m, Fig. 25, is to find two points i and j so located on the profile that the length ij equals the limit of free haul and that the volume of the cut cik within that limit is just sufficient to make the fill cjl. This is easily done by trial when the volume of each cut and each fill in the length am has been computed, and the volumes of the fills have been replaced by the volumes of excavation necessary to make the fills.

If g_1 is the center of gravity of the excavation aik, whose volume is V, and g_2 is the center of gravity of this material deposited in the embankment $jedl$, then $V \times no$ is the haulage of the volume V. But, since this volume must be hauled the length of the free haul ij without extra charge, the overhaul is

$$V \times no - V \times ij = V \times (no - ij) = V \times ni + V \times jo$$

The value of $V \times ni$ is found as follows:

Let x = distance in stations of 100 feet from center of gravity of any prismoid to nearer end, as i, Fig. 25, of length of free haul;

A = area of end of prismoid nearer to point i, in square feet;

A' = area of end of prismoid more remote from point i, in square feet;

p = distance from point i to a point half way between ends of prismoid, in stations of 100 feet;

l = length of prismoid, in stations of 100 feet;

V = volume of prismoid, in cubic yards.

Then, as may be proved by the use of advanced mathematics,

$$x=p+\frac{l}{6}\times\frac{A'-A}{A'+A} \qquad (1)$$

and

$$Vx=V\left(p+\frac{l}{6}\times\frac{A'-A}{A'+A}\right) \qquad (2)$$

When formula 2 is used to find the overhaul of the material from its position in cut to the end i of the limit of free haul, the values to be substituted for the various letters are those which refer to the various prismoids in the cut. However, formula 2 can also be used to find the overhaul $V\times jo$ of the material from the end j of the limit of free haul to its position in the fill, but in this case the values refer to the material in the fill and V represents the volume of excavation necessary to make the fill. The sum of the overhauls for all the prismoids of the cut and the fill is the desired total overhaul.

If a part of the cut is hauled in one direction, and the remainder in the other, the overhaul for each part of the cut must be computed separately.

Example.—The notes for a length of excavation for a railroad are as given in Table IV, Art. 41. In addition, the areas of cross-sections in fill are as follows: Sta. 135, 580 square feet; Sta. 136, 518 square feet; Sta. 137, 768 square feet. The corrected volume of the fill between Stas. 135 and 136 is 2,010 cubic yards and that between Stas. 136 and 137 is 2,368 cubic yards. The length of free haul is 600 feet, extending from Sta. 129 to Sta. 135. The shrinkage of the material placed in embankment is 10 per cent. If 2 cents is paid for each cubic yard overhauled one station, find the total cost of overhaul between Stas. 126 and 137.

Solution.—*Cut.*—In Table VI are recorded in the first column the stations and in the second and third columns the corresponding end areas and volumes of the prismoids, that are given in the sixth and twelfth columns of Table IV.

The numbers in the fourth column of Table VI are the distances in stations of 100 feet from the points half way between the ends of the prismoid to Sta. 129, at which point the free haul begins. Thus, for the excavation between Stas. 126 and 127, the distance p is $129-126.50=2.50$ stations. Similarly, for the prismoid between Stas. 128 +40 and 129, $p=129-128.70=.30$ station.

The value of $\frac{l}{6}\times\frac{A'-A}{A'+A}$ is given for each prismoid in the fifth column.

Thus, for the prismoid between Stas. 126 and 127, the value is $\frac{1.00}{6} \times \frac{157.4-439.4}{157.4+439.4}=-.08$ station, and for the prismoid between Stas. 128 +40 and 129, it is $\frac{.60}{6}\times\frac{724.8-368.5}{724.8+368.5}=+.03$ station.

The numbers in the sixth column are the sums of the corresponding numbers in the fourth and fifth columns; each of these numbers is the distance from Sta. 129 to the center of gravity of the corresponding prismoid, as obtained by formula 1.

Finally, the overhaul for each prismoid is found by multiplying the volume in the third column by the distance x in the sixth column. These results are written in the seventh column. The sum of the numbers

TABLE VI

COMPUTATION OF OVERHAUL IN CUT

Station (1)	End Area Sq. Ft. (2)	Volume V Cu. Yd. (3)	p Sta. (4)	$\frac{l}{6}\times\frac{A'-A}{A'+A}$ (5)	x Sta. (6)	Overhaul Vx (7)
129	368.5					
		1,195	.30	+.03	.33	394
128+40	724.8					
		892	.80	−.01	.79	705
128	483.9					
		1,710	1.50	−.01	1.49	2,548
127	439.4					
		1,074	2.50	−.08	2.42	2,599
126	157.4					
Totals		4,871				6,246

in the seventh column is 6,246 and the overhaul for the cut is, therefore, the equivalent of 6,246 cu. yd. overhauled 1 station.

Fill.—The computations for overhaul for the fill are shown in Table VII. They are similar to those for the cut, but there is one important difference, which calls for the addition of an extra column, the fourth. It is necessary to change the volumes of fill as given in the third column to the volumes of cut required to make the fill. As the shrinkage is 10 per cent., it follows that 100 cu. yd. of cut are necessary to make 90 cu. yd. of fill. Hence the values in column 3 are multiplied by $\frac{100}{90}$ to determine the values in column 4 and these values are used in computing the overhaul.

The total cut required for the fill, which is the sum, 4,864, of the volumes in the fourth column, equals approximately the total cut 4,871 between Stas. 126 and 129. Therefore, tne material excavated between Stas. 126 and 129 will be used between Stas. 135 and 137.

The overhaul for the fill between Sta. 135 and the center of gravity of each prismoid is now computed as shown in Table VII.

TABLE VII

COMPUTATIONS OF OVERHAUL IN FILL

Sta. (1)	End Area Sq. Ft. (2)	Volume of Fill Cu. Yd. (3)	Volume V of Cut Necessary to Make Fill Cu. Yd. (4)	p Sta. (5)	$\frac{l}{6} \times \frac{A'-A}{A'+A}$ Sta. (6)	x Sta. (7)	Overhaul Vx (8)
137	768						
		2,368	2,631	1.50	+.03	1.53	4,025
136	518						
		2,010	2,233	.50	−.01	.49	1,094
135	580						
	Totals	4,378	4,864				5,119

The sum of all the values of Vx for both cut and fill is $6{,}246+5{,}119 = 11{,}365$. This is the equivalent of 11,365 cu. yd. overhauled 1 station; at the rate of 2 cents per cu. yd. per station, the allowance for overhaul is $.02 \times 11{,}365 = \$227.30$. Ans.

EXAMPLES FOR PRACTICE

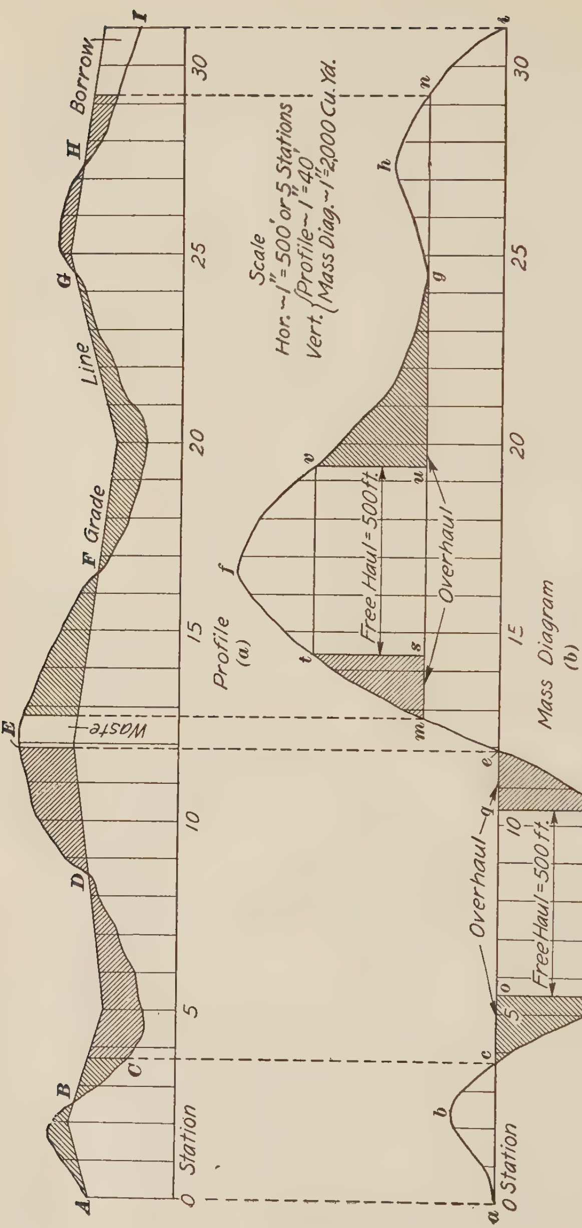
The cross-sectional areas in square feet for a portion of a road are as follows: Sta. 62, cut 210; Sta. 63, cut 284; Sta. 64, cut 323; Sta. 70, fill 235; Sta. 71, fill 330; Sta. 72, fill 89. The limit of free haul extends between Stas. 64 and 70. The amount of cut between Stas. 62 and 63 is 900 cubic yards; the cut between Stas. 63 and 64 is 1,104 cubic yards. The amount of fill between Stas. 70 and 71 is 1,036 cubic yards; the fill between Stas. 71 and 72 is 773 cubic yards. If the allowance for shrinkage is 10 per cent., compute the cost of overhaul between Stas. 62 and 72, when 2 cents is paid for each cubic yard overhauled 100 feet.

Ans. $73.72.

MASS DIAGRAM

75. Introduction.—The method described in the preceding articles for computing the total haulage and the allowance for overhaul is theoretically accurate, provided proper allowances have been made for shrinkage and subsidence. But even the judgment of an experienced engineer cannot always prevent inaccuracies of perhaps several per cent. in his computations, owing to his inability to know beforehand the exact character of the soil to be excavated from a cut and its behavior as to shrinkage. Besides, an unexpected subsidence under a fill may require much more filling material than was calculated and will thus alter to a considerable degree the expected disposition of the excavated material. Consequently, extreme accuracy in haulage computations is not necessary, and for all practical purposes it is sufficient to determine the haulage graphically by means of the mass diagram. This diagram may be plotted by laying off the stations along a horizontal axis and the excess or deficiency of material at each station by an ordinate above or below the axis, as explained in the following articles.

76. Computations for Mass Diagram.—In Fig. 26 (*a*) is a typical profile for a short length of a railroad or a highway, drawn to the horizontal and vertical scales as shown. In Table VIII are given the corresponding volumes between different stations, computed in the manner already explained. The first column contains the stations at which sections are taken; in this table the stations increase downwards, but they may increase upwards or downwards as preferred. The second column contains the cut between each two consecutive stations; for example, the volume of the cut between Stas. 1 and 2 is 305 cubic yards. The third column contains the fill between each two consecutive stations. In the fourth column is the number of cubic yards of excavation needed for the fill given in column 3. This is necessary because, as previously explained, it is customary to pay for overhaul in units of 1 cubic yard of earth in cut hauled one station or 100 feet. As the assumed shrinkage is 10 per cent., 100 cubic yards of excavation make



A
B
C
D
E
F Grade
Line
G
H
I
Borrow
Waste
0 Station
5
10
15
20
25
30
Profile
(a)
Scale
Hor. ~1" = 500' or 5 Stations
Vert. {Profile ~ 1" = 40'
Mass Diag. ~ 1" = 2,000 Cu. Yd.
a
b
c
d
e
f
g
h
i
m
n
o
p
q
r
s
t
u
v
Overhaul
Free Haul = 500 ft.
Mass Diagram
(b)

FIG. 26

but 90 cubic yards of fill; therefore, the volume of excavation necessary to make each fill is $\frac{100}{90}$ times the number of cubic yards given in column 3. For example, the amount of fill between Stas. 3 and 4 is 423, but the cut required for the fill is $\frac{100}{90} \times 423 = 470$ cubic yards, as given in column 4. In the fifth column is given the difference between the total amount of cut and the total amount of material required for the fill from Sta. 0 to the station opposite which the value is given. When the total amount of the cut is greater than the total volume required for the fill, the difference is an excess of cut and is indicated by a plus sign; on the other hand, when the amount of cut is less than the volume required for the fill, the difference is a deficiency of cut or excess of required fill and is indicated by a minus sign.

It is usual in determining the values in column 5 to obtain the excess or deficiency by addition or subtraction from the preceding value. Thus, the excess available at Sta. 2+40 is equal to the excess of 427 at Sta. 2 plus the excess of 28 between Stas. 2 and 2+40, the total excess being $427+28=455$ cubic yards. The amount available at Sta. 3 is the excess of 455 at Sta. 2+40 minus the deficiency of 107 between Stas. 2+40 and 3, or an excess of $455-107=348$ cubic yards. The amount at Sta. 4 is the excess of 348 at Sta. 3 minus the deficiency of 470 between Stas. 3 and 4, and it equals $348-470=-122$ cubic yards. The minus sign indicates that there is not enough cut to make the fill in the interval between Stas. 0 and 4.

In side-hill work, or when there is a transition from cut to fill, some of the cross-sections may be partly in cut and partly in fill. For such cases, there is both cut and fill between two adjacent cross-sections; and the total cut between the given stations is listed in column 2 of Table VIII, the total fill in column 3, and the cut required to supply the material for the fill in column 4. For example, between Stas. 8 and 8+55, there is a cut of 26 cubic yards and a fill of 53 cubic yards, which requires 59 cubic yards of excavated material. The value in column 5 for Sta. 8+55 is found from that at Sta. 8 by adding the value in column 2 and subtracting the volume in column 4; thus, $-2{,}076+26-59=2{,}109$ cubic yards.

TABLE VIII

DETERMINATION OF ORDINATES FOR MASS CURVE

(Material, gravel and clay; shrinkage factor, 10 per cent.)

Sta.	Volumes, in Cubic Yards			
	Cut (+)	Fill (−)	Cut Equivalent of Fill (−)	Total Excess of Cut (+) or Fill (−)
0+00				
1	122			+122
2	305			+427
+40	28			+455
3		96	107	+348
4		423	470	−122
5		713	792	−914
6		510	567	−1,481
7		371	412	−1,893
8		165	183	−2,076
+55	26	53	59	−2,109
9	104	14	15	−2,020
10	512			−1,508
11	733			−775
12	876			+101
13	903			+1,004
14	722			+1,726
15	543			+2,269
16	310			+2,579
+60	172	18	20	+2,731
17	15	41	45	+2,701
18		167	186	+2,515
19		342	380	+2,135
20		408	453	+1,682
21		318	353	+1,329
22		225	250	+1,079
23		146	162	+917
24		89	99	+818
+40	2	20	22	+798
25	42			+840
26	128			+968
27	186	20	22	+1,132
28	24	113	126	+1,030
29		186	207	+823
30		298	331	+492
31		473	524	−32
Total	5,753		5,785	

It is best to find the totals of columns 2 and 4 at the end to verify the work. In this case, the total cut is 5,753, the total fill in terms of equivalent cut is 5,785, and the deficiency is 32 cubic yards, as given in the fifth column opposite Sta. 31 at the end of the length of the line in question.

77. Construction of Mass Diagram.—The mass diagram is laid out, as in Fig. 26 (*b*), directly underneath the profile. First, a horizontal base line is drawn and upon it is marked each station as recorded in Table VIII. Then at each station is plotted the excess of cut given in the table by an ordinate above the base line or the deficiency by an ordinate below the base line. The points so plotted are finally joined by means of a smooth free-hand line and the mass diagram is so completed.

78. Slopes of Mass Diagram.—In the portions of the road where there is a cut, the ordinates of the mass diagram are increasing; hence, a cut is indicated by a line sloping upwards like *ab* or *df*, Fig. 26 (*b*). Where there is a fill, the ordinates are decreasing; hence, a fill is shown by a line sloping downwards like *bd* or *fg*. The high points of the mass diagram, *b*, *f*, and *h*, mark the points *B*, *F*, and *H*, respectively, on the profile where the grade line passes from cut to fill; the low points, *d* and *g*, on the mass diagram mark the points *D* and *G*, respectively, on the profile, where the grade line passes from fill to cut.

In order to economize on construction, the total volume of the earthwork should be kept as small as is practicable. The amount of excess excavation or fill between any two stations is represented by the difference between the ordinates at those two stations. Therefore, as shown in Fig. 26, the steeper slopes represent the heavier cuts and fills and the flat slopes represent the lighter cuts and fills. In general, of two mass diagrams drawn to the same scale, the one with the flatter slopes has the less earthwork and is to be preferred.

79. Haul of Material.—The excess of material is zero at all points where the curve of the mass diagram crosses the base line, such as points *c* and *e*, Fig. 26 (*b*). Between these points the excavation equals the volume required for the fill. Where

the distance between any two points of zero excess is short, the simplest and cheapest way of disposing of the material is to use the cut for the fill between those points. Thus if *ac* and *ce* in view (*b*) are short, it is best to use the cut represented in view (*a*) by *AB* for the fill *BC* and the cut *DE* for the fill *CD*. Therefore, frequent intersections of the base line by the mass curve mean that the material need be carried but a short distance, thus tending to reduce the cost of haulage. In other

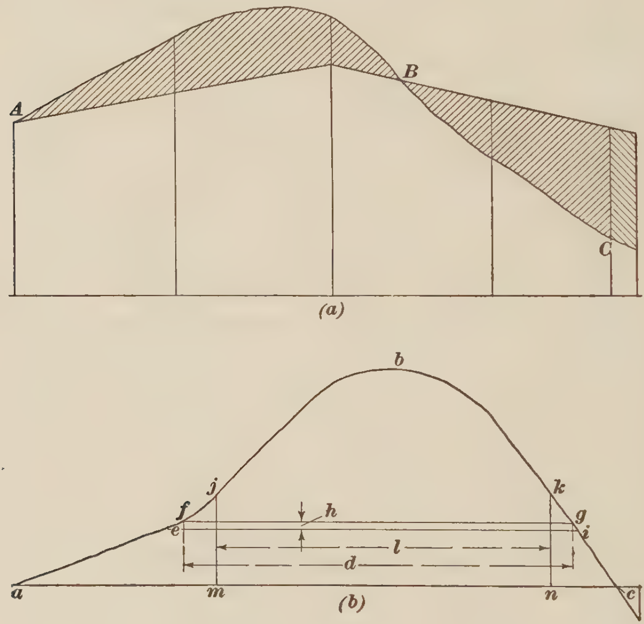


Fig. 27

words, of two mass diagrams, equal in other respects, the one in which the mass curve intersects the base line more frequently is the better.

80. **Measurement of Haul.**—An enlarged portion of the profile of Fig. 26 (*a*) is represented by *ABC* in Fig. 27 (*a*) and the corresponding enlarged portion of the mass diagram is represented by *abc* in view (*b*). In actually moving this earth, it is usual first to throw the material near *B* from cut to fill

and then to work the cut back toward *A*, the material being used to extend the fill toward *C*. Thus, the volume represented by the height *h* in view (*b*) is moved from the location *ef* in cut to *gi* in fill. The actual amount of the haul for this material is the product of the volume *h*, and the distance *d* in stations of 100 feet from the center of *ef* to the center of *gi*, which is represented by the area of the strip *efgi* in the figure. Hence, the area *efgi* represents the haul of the volume *h* and, in the same way, the area *abc* represents the haul of the entire excavation *AB*. If, as in Fig. 26 (*b*), the horizontal scale of the mass diagram is 1 inch = 500 feet, or 5 stations, and the vertical scale for the mass diagram is 1 inch = 2,000 cubic yards, it follows that the scale of area is 1 square inch = $5 \times 2{,}000 = 10{,}000$ cubic yards hauled one station. Therefore, the area of *abc*, in square inches, times 10,000 gives the total amount of haul. In a similar way, the area *cde*, in square inches, multiplied by 10,000, gives the total amount of haul for the material between *C* and *E*.

The area of an irregular figure can be readily measured by a planimeter, and this is the usual method of finding the area of a portion of the mass diagram. The approximate area can also be computed by dividing the figure into triangles and trapezoids, and adding the areas of these parts.

The cost of a job depends to some extent on the haulage. Hence, the mass diagrams for the best designs have a small area; and, of two mass diagrams, drawn to the same scale and equal in other respects, the one with the smaller area is generally the better.

81. Waste and Borrow.—The disposition of the material between *E* and *I*, Fig. 26 (*a*), or between the corresponding points *e* and *i* in the mass diagram in view (*b*), is best studied by first drawing through *g* a horizontal auxiliary base line *mn*. Then the cut between *m* and *f* is carried to the fill between *f* and *g* and the cut between *g* and *h* is moved to the fill between *h* and *n*. There are two ways in which the cut between *e* and *m* and the fill between *n* and *i* may be handled. The first method is to haul the material represented by *em* to *ni* and use it for fill. The second method is to throw away, or *waste*, the cut

between e and m and borrow the material for the fill between n and i from some convenient place. In the mass diagram, the latter method is indicated for the sake of illustration. The choice between these two methods is usually determined by the cost. The first involves one excavation and a large amount of overhaul; the second requires two excavations, usually within the limits of free haul. In the present case, the average length of excess haul is about 1,800−500 or 1,300 feet.

82. Computation of Overhaul.—Suppose it is desired to find the overhaul between A and C, Fig. 27 (a), from the mass diagram in view (b), when the limit of free haul is the distance l. Two points l feet apart are first laid off to scale on a straightedge, and this edge, always kept parallel to the base line, is moved until both marked points are on the mass curve. The points j and k thus located are at the same height above the base line, and hence the cut and fill between j and k balance. As all hauling between j and k is within the distance l, the material must be hauled free. Then the area ajm plus the area knc represents the overhaul. If the horizontal scale is 1 inch = 1 station and the vertical scale is 1 inch = 400 cubic yards, the combined area in square inches multiplied by 400 gives the overhaul.

EXAMPLE.—Compute the overhaul for the profile and mass diagram of Fig. 26 when the material em is wasted, the material ni is borrowed, and the limit of free haul is 500 feet.

SOLUTION.—The overhaul is represented in view (b) by the shaded portions, whose areas as measured by a planimeter are as follows:

$$\begin{aligned} \text{area } cop &= .101 \text{ sq. in.} \\ \text{area } eqr &= .087 \text{ sq. in.} \\ \text{area } mst &= .103 \text{ sq. in.} \\ \text{area } guv &= .198 \text{ sq. in.} \\ \hline \text{Total} &= .489 \text{ sq. in.} \end{aligned}$$

The scale of the mass diagram in Fig. 26 (b) is 1 sq. in. = 10,000 cu. yd. hauled one station; therefore, the total overhaul is .489×10,000 = 4,890 cu. yd. hauled one station. Ans.

83. Uses of Mass Diagram.—The mass diagram serves several useful purposes:

(1) It suggests changes in the grade line which lessen the cost of the earthwork and, hence, reduce the cost of construction. This purpose may be best fulfilled by studying together a plan, a profile, and a mass diagram of the proposed road. The grades should be so arranged that the curves of the mass diagram are as flat as possible, in order to lessen the amount of the excavation; also, an endeavor should be made to arrange the grades so as to lessen the areas which represent haul.

(2) The mass diagram when prepared for several possible lines shows clearly which is the most economical. Frequently, there are several practicable locations for a railroad or a highway. The best method of comparison is the preparation and study of a plan, profile, and mass diagram for each line.

(3) The mass diagram enables the contractor to understand just what work he will be expected to do and, therefore, helps him decide in what manner and with what kind of equipment he can perform the work most economically.

(4) It assists in the determination of the amount of the overhaul.

DESIGN OF RETAINING WALLS

Serial 4067 Edition 1

STABILITY OF RETAINING WALLS

INTRODUCTION

DEFINITIONS

1. Kinds of Walls.—A *retaining wall* is a wall employed to hold in place a filling of loose earth, sand, gravel, cinders, or similar material, deposited behind the wall after it is built. In Fig. 1 is shown a cross-section of a retaining wall *a* holding back a fill behind it.

Sometimes, a wall is erected against the vertical face of an excavation, in which case it is called a *face wall*. The earth behind a face wall, being in its natural condition, is firm and solid, and is not so likely to slide or cave as the filling placed behind a retaining wall. Hence, a face wall can usually be made thinner than a retaining wall.

In areaways and cellars, the earth is retained by means of walls that are braced at the bottom and top by floors; such walls are known as *braced walls*.

2. Parts of Retaining Wall.—The outer, or exposed, side of a retaining wall is known as the front, or face, and the side that is in contact with the filling as the back of the wall. The bottom of a retaining wall is called the base or foot of the wall. The outer and inner edges of the base are known, respectively, as the *toe* and *heel* of the wall. In Fig. 1, the toe of the wall is at *b* and the heel at *c*. A retaining wall is usually

constructed with a foundation course, or footing, as *d* in the illustration, which is treated as a part of the wall.

The top of the wall is generally provided with a coping, which projects from 3 to 6 inches beyond the face of the wall and is from 12 to 24 inches high. In low walls of a height of 4 feet or less, the coping is frequently omitted.

3. Surcharge.—The surface of the fill behind a retaining wall may be level or it may slope upwards or downwards from the wall. When the fill slopes upwards, the part of the fill above the level of the top of the retaining wall is called surcharge. In Fig. 1, the earth *efgh* above the level *eh* is the surcharge. When the fill behind a wall is level with the top of the wall, there is really no surcharge, but it is often said that the wall has a horizontal surcharge. Also, when the surface of the fill slopes downwards, the wall is said to have a negative surcharge.

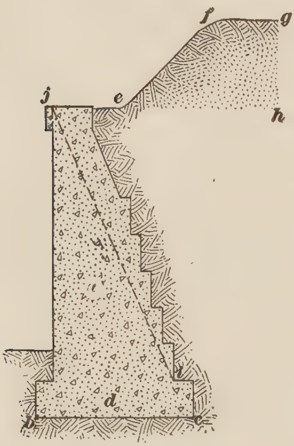


Fig. 1

The term loaded surcharge is used to designate any live load on the surface of the fill that may increase the pressure exerted by the fill against the wall. For example, where a railroad track is laid on the fill behind a wall, the weight of a train on the track is a surcharge on the wall.

PHYSICAL PROPERTIES OF EARTHS

4. General Remarks.—Before the design of a retaining wall is started, it is necessary to know certain physical properties of the filling material that will be put behind the wall and of the foundation soil on which the wall will rest. The information in which the designer is mainly interested is the angle or slope of repose and the weight per unit volume of the filling material, and the crushing strength of the foundation soil.

5. Angle of Repose.—The angle of repose of a granular material is the angle with the horizontal at which the material will stand without sliding when it is loosely piled. For a material such as dry, clean sand, the slope of repose or angle of repose may be determined experimentally by the method illustrated in Fig. 2. The material is allowed to drop from a funnel through a small opening a onto a horizontal table until a cone about 12 inches high at the center is formed. The height h of the cone divided by the horizontal distance r is the slope of repose, or the tangent of the angle of repose. If six or more independent measurements of the distance r to various

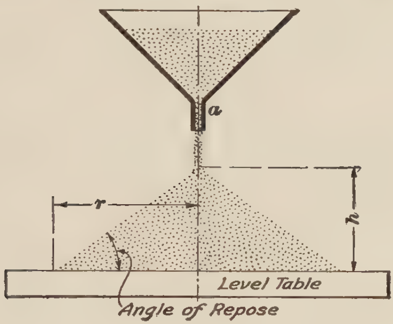


FIG. 2

points around the perimeter of the base of the cone are taken, the average value is sufficiently accurate.

The angle of repose varies greatly with the different soils, and also with soils of the same kind containing different amounts of moisture. Roots affect the cohesiveness of the soil, because they serve as binders. Perfectly dry, clean sand, which has no appreciable cohesion or tenacity between the particles, gives a fairly uniform experimental value of the angle of repose, but other materials give varying results. With the exception of clay, slight dampness in a material increases the angle of repose, and wetness decreases it. Even a small amount of moisture lowers the angle of repose of clay.

Experience has shown that nearly all soils used for the filling behind a retaining wall stand, when properly drained, at a slope of $1\frac{1}{2}$ horizontal to 1 vertical or at an angle of 33° 41′.

However, some soils, such as fragments of rock or dry sand, stand at a slope of 1 to 1, or at an angle of 45°, and this value may be used for such material. If, on the other hand, it is necessary to use a filling material of semifluid character, which will not stand even at a slope of $1\frac{1}{2}$ horizontal to 1 vertical, its angle of repose should be determined experimentally rather than taken from a table of values.

6. Weight of Filling Material.—All filling materials weigh more when wet than when dry. Furthermore, every retaining wall is at some time subjected to the pressure of water-soaked filling. Hence, in designing a retaining wall, the weight of wet filling material should be considered. Water-soaked filling, of earthy character, weighs from 115 to 125 pounds per

TABLE I

SAFE LOADS ON EARTH FOUNDATION BEDS

Kinds of Material	Loads in Tons per Square Foot
Hard rock	40
Medium rock	25
Hardpan	10
Soft rock	8
Gravel	6
Sand, firm and coarse	4
Clay, hard and dry	3
Sand, fine and dry	3
Ordinary firm clay	2
Sand and clay, mixed or in layers	2
Sand, wet	2
Clay, soft	1

cubic foot. The weight of gravel is variously placed between 100 and 135 pounds per cubic foot. Broken stone is generally assumed to weigh a few pounds less than gravel, but the range is wide. The filling actually used is frequently of mixed character, and its average unit weight can be determined accurately only by weighing numerous samples and

estimating the relative proportions of each kind of material. Unless the weight of filling is definitely known, it is advisable to use an ample value.

7. **Crushing Strength of Subsoil.**—The design of every retaining wall should include an investigation of the pressure on the natural subsoil on which the artificial structure is built. Soils vary gradually from hard bed rock, which can safely carry any masonry structure, to soft mud or quicksand, which is virtually a liquid and has a very low bearing capacity. There are various tables which attempt to classify soils as to bearing power and credit each class with a certain safe unit load. A comparison of these tables shows a wide range of values for soils which are given substantially the same name. This indicates the uncertainty in the classification. Therefore, Table I, which has been compiled from various sources, should be used only as a guide in selecting the safe unit value that can be applied to the soil for the particular problem under discussion.

POSSIBLE FAILURES OF RETAINING WALLS

8. **Types of Possible Failures.**—The fill behind a retaining wall exerts a pressure against the back of the wall, and if the wall is not properly designed, that pressure may cause the wall to overturn, to slide forwards, or to settle at its toe. In some cases, the wall may shear on a plane between its top and bottom, or the toe of the wall may be crushed on account of the great pressure concentrated there.

9. **Overturning.**—The most common cause of failure of retaining walls is overturning. In Fig. 3 (*a*) is shown a cross-section of a typical concrete wall under the action of three forces: (1) the earth pressure, represented by the resultant force P, which is assumed to be horizontal; (2) the weight of the wall, which may be treated as a single force W acting through the center of gravity of the masonry; and (3) the weight of the filling material resting on the wall, which may be denoted by the force W_f acting through the center of gravity of the portion of the fill between the back of the wall and the

vertical plane bc through the heel. Then, the force that tends to overturn the wall about the toe a is the horizontal earth pressure P, and the forces that resist this overturning are the vertical forces W and W_f. In order that the wall may be prevented from overturning, the sum of the moments of the vertical forces must exceed the moment of the horizontal force. In other words, $W \times ad + W_f \times ae$ must be greater than $P \times af$.

When the earth pressure P is inclined, as in (b), it is assumed for convenience to be replaced by its horizontal component P_h and its vertical component P_v at the point i where it intersects

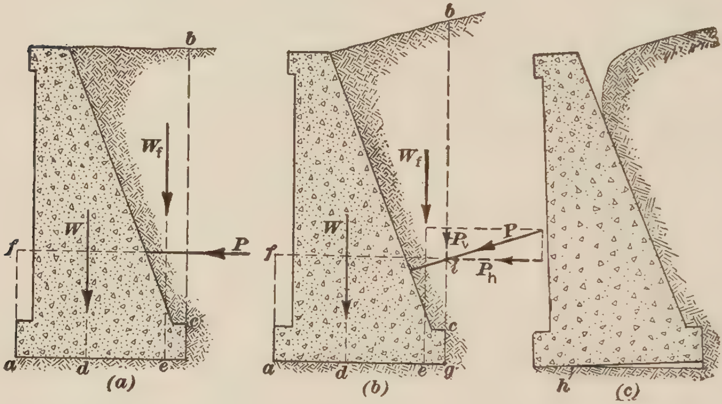


FIG. 3

the vertical plane bc through the heel. In this case, the overturning force is the horizontal component P_h and the forces resisting overturning are the vertical forces W, W_f, and P_v. To prevent overturning, $W \times ad + W_f \times ae + P_v \times ag$ must exceed $P_h \times af$.

The soil on which the retaining wall rests and, to some extent, the material of which the wall is constructed yield under the great pressure produced at the toe. As a result, the point about which overturning tends to take place is not exactly at the toe but at an appreciable distance in back of it, as point h in (c), where the conditions are shown exaggerated. It is evident from a study of (a) and (b) that the lever arm of the horizontal force about any point on the base is the same,

whereas the lever arms of the vertical forces are less about a point in back of a than about a. Hence, it is not safe to design a wall so that the sum of the moments of the vertical forces is but slightly greater than the moment of the horizontal force, because the actual moments of the vertical forces are materially less than the corresponding values with respect to the toe. However, since the point about which overturning actually tends to take place is not known, it is convenient to determine the moments with respect to the toe and so to proportion the section of the wall that the sum of the moments of the vertical forces is considerably greater than the moment of the horizontal force.

10. Sliding on Foundation.—A retaining wall seldom fails by sliding on its foundation if reasonable care is taken in its construction. Theoretically, such a failure occurs when the horizontal component of the pressure of the fill exceeds the product of the sum of the vertical forces acting on the wall and the coefficient of friction for the material of the wall and that of the foundation. In a well-constructed wall, the foundation is purposely left rough so as to increase the friction between the bottom of the wall and the foundation soil. Furthermore, the wall cannot slide without displacing the earth that is generally in front of it. Nevertheless, where there is any doubt as to the stability of a wall against sliding, the wall should be built with one or more keys, or projections, into the foundation soil, in order to increase the resistance to sliding.

11. Crushing at Toe.—Retaining walls of great height or those resting on relatively weak foundation soils may settle at the toe. When an inferior quality of material is used for a high wall resting on a firm soil, the masonry at the toe may crush because of the high compressive stress at that point.

EARTH PRESSURE

MAIN THEORIES

12. Nature of Problem.—The first step in the design of a retaining wall, or in the investigation of the stability of a wall already built, is the determination of the amount, direction, and point of application of the resultant pressure exerted by the filling on the back of the wall. The usual filling material is neither a liquid nor a solid, but is a granular material with a certain amount of cohesion between its particles. This cohesive action has an important effect on the pressure exerted against the wall, and therefore variations in the condition of the fill cause changes in the pressure. When the filling is first deposited behind a wall, it exerts a certain pressure. As time goes on, it becomes consolidated, the cohesion between the particles is increased, and the pressure on the wall tends to decrease. On the other hand, saturation with water or vibration due to loads moving over the fill may reduce the cohesion in the filling and so increase the pressure on the wall. Therefore, an accurate determination of the amount, direction, and point of application of the resultant earth pressure on a retaining wall would require extensive preliminary study and experiment.

13. General Assumptions.—Many theories concerning earth pressure have been evolved, nearly all of them being based on the following assumptions:

1. The material of the filling is a homogeneous mass consisting of grains which have resistance to rolling over each other.
2. The pressure of the filling acts in a definite direction that can be determined.
3. The location of the resultant pressure on the back of the wall is the same as for liquid pressure.

The principal theories in common use in the design of retaining walls are those advanced by Rankine and Coulomb, and modifications of these theories, and the equivalent-fluid-pressure method.

14. Rankine's Theory.—In Rankine's theory, the filling is assumed to be an incompressible, homogeneous, granular mass without cohesion, the particles being held in place by their resistance to rolling over each other. The amount and direction of the earth pressure are determined by considering the stresses on a unit volume in the retained mass of earth, and applying the principles of elasticity of materials. According to this theory, the direction of the earth pressure on a vertical wall is parallel to the surface of the fill.

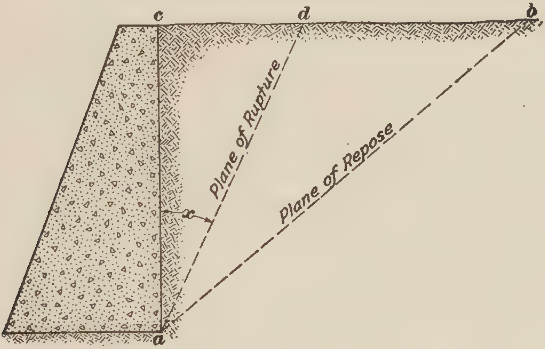


FIG. 4

15. Coulomb's and Other Wedge Theories.—In the wedge theory of earth pressure, which was probably first proposed by Coulomb, it is assumed that the earth behind a retaining wall tends to slide along an inclined plane, known as the plane of rupture, and that the maximum pressure on the wall is exerted by the wedge-shaped prism of earth which is bounded by the back of the wall, the plane of rupture, and the surface of the filling. In Fig. 4, the back of the wall is vertical, the surface of the fill is level, and *ab* represents the plane of repose of the filling. For these conditions, it is assumed that the prism of earth causing maximum pressure is bounded by the back *ac* of the wall and the plane of rupture *ad*, which is so located that the angle *cad* or *x* bisects the angle *cab*. When the back of the wall is sloped or when there is a surcharge, the plane of rupture is not midway between the plane of repose and a vertical plane

through the heel of the wall, its position depending on the conditions.

The direction of the earth pressure in Coulomb's theory is assumed to be perpendicular to the back of the wall; in other words, the friction between the filling and the back of the wall is neglected. However, many designers employ a modification of Coulomb's theory in which account is taken of this friction, and it is assumed that the angle between the line of action of the resultant earth pressure and the perpendicular to the back of the wall is equal to the angle of friction between the material of the filling and that of the wall. The value of this angle is as low as 18° for moist clay and as high as 37° for gravel or broken stone.

16. Equivalent Fluid Pressure.—In the equivalent-fluid-pressure method, it is assumed that the pressure exerted by the filling against the back of a retaining wall is equivalent to the pressure caused by a liquid whose weight is some fraction of the weight of the filling. The value of the fraction is chosen so that the overturning moment due to the equivalent fluid pressure is the same as that due to the pressure obtained when Rankine's or Coulomb's theory is assumed. Of course, in this method of design, the pressure of the filling is assumed to be horizontal.

DETERMINATION OF PRESSURE

17. Walls With Vertical Back and Horizontal Surcharge. Rankine's theory is the one most generally used in the design of retaining walls, and therefore the methods and formulas given here are based on that theory. When the back of the wall is vertical and there is a horizontal surcharge, as in Fig. 5 (a), the earth pressure on any part of the wall is assumed to be horizontal and its value is assumed to vary at a uniform rate from zero at the top of the fill to a maximum at the bottom of the wall. The intensity of pressure at any depth can then be computed by the formula

$$p_x = w h_x \frac{1-\sin Z}{1+\sin Z} \qquad (1)$$

in which p_x = pressure, in pounds per square foot, at any depth;

w = weight of filling material, in pounds per cubic foot;

h_x = depth, in feet, from top of fill to point under consideration;

Z = angle of repose of filling material.

The distribution of the unit pressure is represented by the triangle *abc*; the pressure at the bottom of the wall, indi-

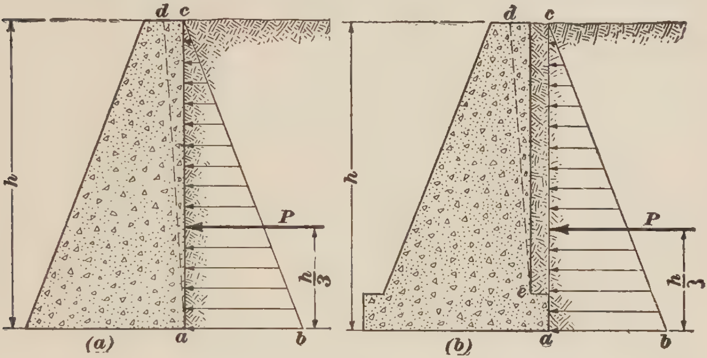


FIG. 5

cated to a convenient scale by the base *ab* of this triangle, is $p = wh\frac{1-\sin Z}{1+\sin Z}$. Hence, the total resultant pressure per foot of wall, which is represented by the area of the pressure triangle, may be computed by the formula

$$P = \tfrac{1}{2}wh^2\frac{1-\sin Z}{1+\sin Z} \qquad (2)$$

in which P = total horizontal pressure per foot of length of wall, in pounds;

h = height of wall, in feet;

w and Z have the same meanings as in formula 1.

By the relations among trigonometric functions, formula 2 may be reduced to the following form:

$$P = \tfrac{1}{2}wh^2 \tan^2\left(45° - \frac{Z}{2}\right) \qquad (3)$$

Since the resultant earth pressure passes through the center of gravity of the triangle *abc*, it may be assumed to act at a distance from the base of the wall equal to one-third of the height of the wall, as shown in (*a*).

In the case of a wall with a footing that projects at the heel, like that shown in view (*b*), the filling in the rectangle directly over the projection at the heel is included as a part of the wall when the pressure on the wall is considered, and the vertical plane represented by *ac* is treated as the back of the wall.

When the slope of repose of the material is taken as $1\frac{1}{2}$ horizontal to 1 vertical, $Z=33°\ 41'$ and

$$P=.143wh^2 \qquad (4)$$

For a material whose slope of repose is 1 to 1, in which case $Z=45°$,

$$P=.086wh^2 \qquad (5)$$

EXAMPLE.—A retaining wall with a vertical back is 16 feet high. The filling material weighs 115 pounds per cubic foot and has an angle of repose of 40°. If the surface of the fill is level, what is the resultant pressure on the back of the wall?

SOLUTION.—Here, $w=115$ lb. per cu. ft., $h=16$ ft., and $Z=40°$. By formula 2,

$$P=\tfrac{1}{2}wh^2\frac{1-\sin Z}{1+\sin Z}=\tfrac{1}{2}\times 115\times 16^2\times\frac{1-.643}{1+.643}=3{,}200 \text{ lb. Ans.}$$

18. Walls With Vertical Back and Sloping Surcharge.—In Fig. 6 (*a*) is represented a surcharged wall with a vertical back and no footing; the angle X between the surface of the embankment and the horizontal is less than the angle of repose of the filling. In this case, the earth pressure at each point is assumed to be parallel to the surface of the fill and the intensity of pressure at any depth is found by the formula

$$p_x=wh_x\cos X\frac{\cos X-\sqrt{\cos^2X-\cos^2Z}}{\cos X+\sqrt{\cos^2X-\cos^2Z}} \qquad (1)$$

in which X = angle that surface of fill makes with the horizontal; p_x, w, h_x and Z have the same meanings as in the preceding article.

The distribution of pressure is represented by the triangle *abc* in which the base *ab* indicates, to scale, the unit pressure at the bottom of the wall where $h_x = h$. The resultant pressure P per foot of length of wall may be computed by the formula

$$P = \tfrac{1}{2} w h^2 \cos X \frac{\cos X - \sqrt{\cos^2 X - \cos^2 Z}}{\cos X + \sqrt{\cos^2 X - \cos^2 Z}} \qquad (2)$$

and its point of application is located along the back of the wall at a distance above the base equal to one-third of the height of the wall.

When a surcharged wall has a footing, as in (*b*), the vertical plane represented by *ac* is treated as the back of the wall, and

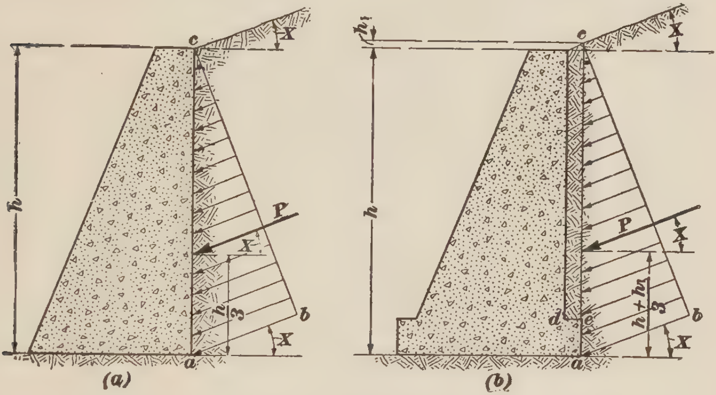


Fig. 6

the vertical distance along *ac* from the base of the footing to the point of application of the earth pressure is one-third of the height *ac*. The magnitude of the resultant pressure per foot of wall is

$$P = \tfrac{1}{2} w (h + h_1)^2 \cos X \frac{\cos X - \sqrt{\cos^2 X - \cos^2 Z}}{\cos X + \sqrt{\cos^2 X - \cos^2 Z}} \qquad (3)$$

in which h_1 is the vertical distance from the level of the top of the wall to the point where the vertical line through the heel of the wall intersects the surface of the fill.

When a wall is surcharged, it is customary to have the surface of the fill slope at the angle of repose, if possible.

For this condition, formula 2 for a wall without a footing reduces to

$$P = \tfrac{1}{2} w h^2 \cos Z \qquad (4)$$

and formula 3 for a wall with a footing becomes

$$P = \tfrac{1}{2} w (h + h_1)^2 \cos Z \qquad (5)$$

EXAMPLE.—A retaining wall like that shown in Fig. 6 (*b*) holds back an embankment whose surface is inclined at the slope of repose, which is $1\frac{1}{2}$ horizontal to 1 vertical. The height h of the wall is 20 feet, the projection de of the footing is 2 feet, and the filling weighs 110 pounds per cubic foot. Find (*a*) the resultant pressure on a foot length of the wall and (*b*) the distance from the heel a to the point where the line of pressure cuts the vertical ac.

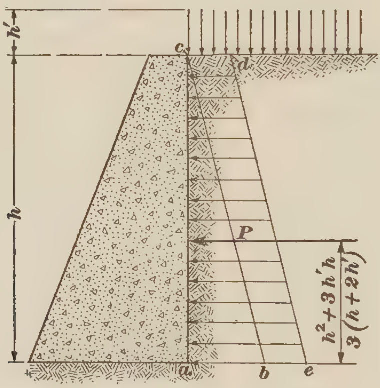


FIG. 7

SOLUTION.—(*a*) In this case, $w = 110$ lb. per cu. ft., $h = 20$ ft., Z is the angle whose tangent is $1 \div 1\frac{1}{2}$, or $Z = 33° 41'$, and $h_1 = dc \tan Z = 2 \times \frac{2}{3} = 1.33$ ft. Then, by formula 5,

$$P = \tfrac{1}{2} w (h + h_1)^2 \cos Z = \tfrac{1}{2} \times 110 \times 21.33^2 \times .832 = 20{,}800 \text{ lb.} \quad \text{Ans.}$$

(*b*) The height ac is $h + h_1 = 21.33$ ft., and the distance from a to the point of application of P is $\frac{1}{3} ac = \frac{1}{3} \times 21.33 = 7.11$ ft. Ans.

19. Walls With Vertical Back and Uniform Load on Filling.—When a level fill behind a wall carries a superimposed uniform load, as in Fig. 7, the pressure against the wall due to the superimposed load is added to the pressure caused by the fill itself. The unit pressure at any point exerted by the

fill alone is computed by formula 1, Art. 17, and the distribution of this unit pressure is represented by the triangle *abc*. Since the surcharge in this case is uniformly distributed over the surface of the fill, it is assumed that the horizontal pressure on the wall due to the surcharge is also uniform. Thus, the distribution of the unit pressure exerted by the surcharge is represented by the parallelogram *bcde*. The amount of the unit pressure due to the superimposed load may be computed by the formula

$$p' = w' \frac{1 - \sin Z}{1 + \sin Z} \qquad (1)$$

in which p' = unit pressure, in pounds per square foot, at any point due to loaded surcharge;
w' = superimposed load, in pounds per square foot;
Z = angle of repose of filling material.

It is often convenient to treat the superimposed load as an equivalent height of fill. The effective height h' of fill, in feet, that is equivalent to the surcharge may be found by dividing the superimposed load w', in pounds per square foot, by the weight w of the filling material, in pounds per cubic foot, or $h' = w' \div w$. Then formula 1 becomes

$$p' = wh' \frac{1 - \sin Z}{1 + \sin Z} \qquad (2)$$

The magnitude of the resultant pressure P per foot of wall is

$$P = \tfrac{1}{2} wh(h + 2h') \frac{1 - \sin Z}{1 + \sin Z} \qquad (3)$$

and the vertical distance y from the base of the wall to the point of application of the resultant pressure is

$$y = \frac{h^2 + 3h'h}{3(h + 2h')} \qquad (4)$$

EXAMPLE.—If the filling in the example of Art. 17 supports a uniform superimposed load of 400 pounds per square foot, find (*a*) the resultant pressure per foot length of wall and (*b*) the distance from the base to the line of action of the pressure.

SOLUTION.—(*a*) In this example, $w=115$ lb. per cu. ft., $w'=400$ lb. per sq. ft., $h'=\frac{w'}{w}=\frac{400}{115}=3.48$ ft., $h=16$ ft., and $Z=40°$. Hence, by formula 3,

$$P=\tfrac{1}{2}wh(h+2h')\frac{1-\sin Z}{1+\sin Z}=\tfrac{1}{2}\times115\times16\times(16+2\times3.48)\times\frac{1-.643}{1+.643}$$

$$=4{,}590 \text{ lb. Ans.}$$

(*b*) By formula 4,

$$y=\frac{h^2+3h'h}{3(h+2h')}=\frac{16^2+3\times3.48\times16}{3\times(16+2\times3.48)}=\frac{423}{68.9}=6.14 \text{ ft. Ans.}$$

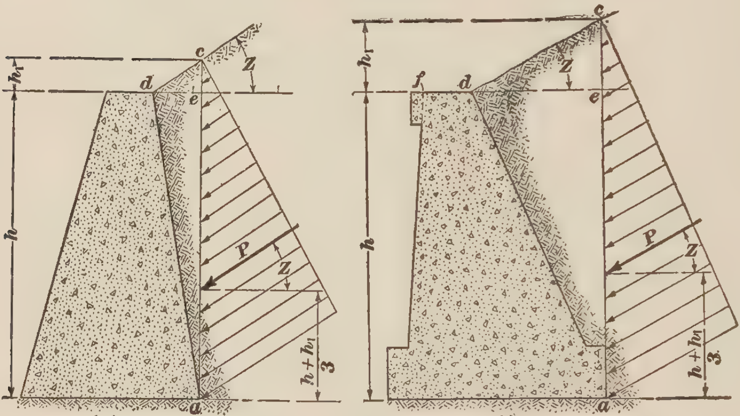


FIG. 8

20. Walls With Sloping Back.—The lateral pressure on a wall with horizontal filling is assumed to be the same whether the back of the wall is vertical or slopes forwards, or away from the filling. Thus, the principles of Art. 17 also apply without modification to a wall whose back is represented by the dashed line *ad* in Fig. 5 (*a*), or by the dashed line *ed* in (*b*). Similarly, the formulas of Art. 19 apply to a wall whose back slopes forwards.

When a wall retains a fill with a sloping surcharge and the back of the wall slopes forwards, as in Fig. 8, the resultant pressure against the back of the wall is determined by formula 3 or 5, Art. 18, whether the wall has a footing or not; also, the

point of application of the pressure is located along the vertical *ac* at a distance from the base equal to one-third of the height *ac*. In Fig. 8 (*a*) or (*b*), h_1 is equal to *de* tan *Z*. If a drawing of the wall section is made to scale, the height h_1 can be measured.

21. Allowance for Track Loads on Fill.—The American Railway Engineering Association recommends that, when the

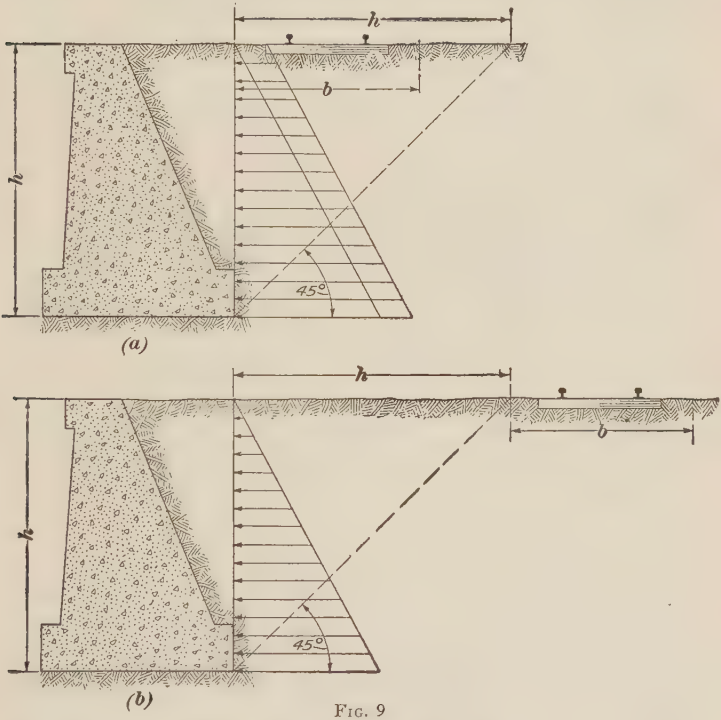


FIG. 9

filling behind a retaining wall carries one or more railroad tracks, the track loads should be distributed as follows: For a single track the entire load should be considered uniformly distributed over a width of 14 feet; for tracks spaced 14 feet or more on centers, the load on each track should be distributed over a width of 14 feet; and for tracks spaced less than 14 feet on centers, the load on each track should be dis-

tributed over a width equal to the distance between centers of tracks. Where the nearer edge of the loaded width b is vertically above the heel of the wall, as in Fig. 9 (a), or inside the vertical line through the heel, the full effect of the superimposed load is to be assumed in calculating the pressure on the wall. On the other hand, where the nearer edge of the loaded width b is at a distance from the heel greater than the height h of the wall, as in (b), the effect of the superimposed load may be neglected. In all cases where the nearer edge of the loaded width is between the positions shown in (a) and (b), and at a horizontal distance m from the heel of the wall, the part of the load to be considered is found by multiplying the entire load by the ratio $\frac{h-m}{h}$.

EXAMPLE.—An embankment for a single track carrying a load of 8,000 pounds per foot of length is retained by a wall 18 feet high, the horizontal distance from the heel of the wall to the center of the track being 12 feet. Find the load w' per foot to be considered as surcharge on the wall.

SOLUTION.—Since the loaded width of the embankment for a single track is taken as 14 ft., the distance from the center line of the track to the edge of the loaded width is 7 ft., and the distance from the heel of the wall to the nearer edge of the loaded width is $m=12-7=5$ ft. Also, the total load per sq. ft. on the filling is $\frac{8,000}{14}=571$ lb. Hence,

$$w'=\frac{h-m}{h}\times 571=\frac{18-5}{18}\times 571=412 \text{ lb. per sq. ft. Ans.}$$

22. Allowance for Other Loads on Fill.—The fill in back of a retaining wall may support not only railroad tracks but also various other kinds of structures, such as a street or highway, or a footing of a building. For these structures the load should be considered distributed over the entire bearing width. Thus, the loaded width of the footing for the building wall in Fig. 10 is the width b of the footing. As in the case of track loads, the amount of the superimposed load to be assumed effective in exerting pressure on the retaining wall depends on the horizontal distance from the heel of the wall to the nearer edge of the loaded width.

When the superimposed load is applied below the top of the retaining wall, as in Fig. 10, only the lower part of the wall is subjected to pressure due to the load. The distribution of unit pressure exerted by the filling material is represented by the triangle *acd*, and the pressure due to the superimposed load is represented by the parallelogram *defg*. The amount and point of application of the total resultant pressure are determined by combining the resultant pressure due to the earth and that due to the load, as in the following example.

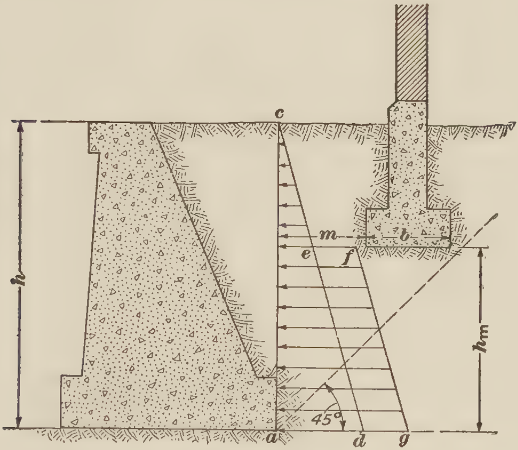


FIG. 10

EXAMPLE.—In Fig. 10, the height h of the retaining wall is 14 feet, the weight of the filling material is 115 pounds per cubic foot, and its slope of repose is $1\frac{1}{2}$ horizontal to 1 vertical. The building wall carries a load of 18,000 pounds per foot of length; the width b of its footing is 3 feet; the distance m from the nearer edge of this footing to the heel of the retaining wall is 6 feet; and the difference in elevation, h_m, between the bottoms of the footings is 8.5 feet. Find (a) the total resultant horizontal pressure on the back of the wall per foot of length and (b) the height of its line of action above the bottom of the retaining-wall footing.

SOLUTION.—(a) By formula 4, Art. 17, the resultant pressure per foot of wall due to the weight of the filling alone is

$$P = .143wh^2 = .143 \times 115 \times 14^2 = 3{,}220 \text{ lb.}$$

The superimposed load per sq. ft. from the building is $18{,}000 \div 3 = 6{,}000$ lb. and

$$w' = \frac{h_m - m}{h_m} \times 6{,}000 = \frac{8.5 - 6}{8.5} \times 6{,}000 = 1{,}770 \text{ lb. per sq. ft.}$$

Then, by formula 1, Art 19,

$$p' = w' \frac{1 - \sin Z}{1 + \sin Z} = 1{,}770 \times \frac{1 - .555}{1 + .555} = 507 \text{ lb. per sq. ft.}$$

and the resultant pressure due to the surcharge is $P' = p'h_m = 507 \times 8.5 = 4{,}310$ lb.

Hence, the total resultant pressure is $P + P' = 3{,}220 + 4{,}310 = 7{,}530$ lb. Ans.

(*b*) The distance from the bottom of the retaining-wall footing to the resultant earth pressure is $\frac{1}{3} \times 14 = 4.67$ ft. and to the resultant pressure due to the superimposed load is $\frac{1}{2} \times 8.5 = 4.25$ ft. Hence, the moment of the earth pressure about the bottom of the footing is $3{,}220 \times 4.67 = 15{,}000$ ft.-lb.; that of the resultant pressure due to the superimposed load is $4{,}310 \times 4.25 = 18{,}300$ ft.-lb.; the total resultant moment is $15{,}000 + 18{,}300 = 33{,}300$ ft.-lb.; and the distance from the bottom of the footing to the line of action of the total resultant pressure is $33{,}300 \div 7{,}530 = 4.4$ ft. Ans.

EXAMPLES FOR PRACTICE

1. The filling behind a retaining wall with a vertical back weighs 100 pounds per cubic foot and has an angle of repose of 35°. If the surface of the fill is level with the top of the wall and the height of the wall is 14 feet, what is the resultant pressure per foot of length of wall?

Ans. 2,650 lb.

2. If the filling in example 1 carries tracks 13 feet on centers, the center of the nearest track being 10 feet from the heel of the wall, and the load per foot on each track is 8,000 pounds, find (*a*) the load per square foot to be considered as surcharge on the wall; (*b*) the resultant pressure per foot of wall; and (*c*) the vertical distance from the base of the wall to the line of action of the pressure.

Ans. $\begin{cases} (a) \text{ 461 lb. per sq. ft.} \\ (b) \text{ 4,400 lb.} \\ (c) \text{ 5.59 ft.} \end{cases}$

3. A wall like that shown in Fig. 8 (*b*) has a vertical face and a sloping back; if the projection of the coping is not considered, the thickness *df* of the wall at the top is 2 feet, the thickness just above the footing is 6 feet, the projections at the heel and toe are each 1.5 feet, the thickness of the footing is 3 feet, and the total height *h* is 15 feet. The filling weighs 120 pounds per cubic foot and the upward slope of

its surface, which is equal to the slope of repose, is $1\frac{1}{2}$ horizontal to 1 vertical. Find (*a*) the vertical distance h_1, (*b*) the resultant pressure P per foot of length of wall; and (*c*) the vertical distance along *ac* from the heel of the wall to the point of application of the pressure.

$$\text{Ans.}\begin{cases}(a)\ 3.67\text{ ft.}\\(b)\ 17{,}400\text{ lb.}\\(c)\ 6.22\text{ ft.}\end{cases}$$

INVESTIGATION OF STABILITY

STABILITY AGAINST OVERTURNING

23. Conditions of Stability.—Stability against overturning demands that the sum of the moments of the vertical forces acting on a retaining wall about the toe of the wall should be greater than the moment of the horizontal component of the earth pressure. However, in the usual investigations of retaining walls, it is customary simply to locate the point where the resultant of all forces acting on the wall cuts the base. The distance of that point from the toe of the wall indicates not only the stability of the wall against overturning, but also the distribution of the pressure on the foundation, which is of prime importance in the design and investigation of retaining walls.

When the resultant passes through the toe of the wall, the total moment of the vertical forces is equal to the moment of the horizontal forces. Hence, in all cases where the resultant passes in back of the toe, the sum of the moments of the vertical forces is greater than the moment of the horizontal forces. If the wall rests on a firm soil, it is considered to be stable when the resultant cuts the base within the middle third. In the case of a wall resting on a rock foundation, the resultant may even pass slightly outside the middle third. However, for a wall resting on a soft soil, the resultant should cut the base near the center so that the wall will not tend to tip outwards at all.

24. Moment of Resultant.—In order to locate the point where the resultant of all the forces acting on a wall cuts the base, it is necessary first to compute the moment of that resultant about the toe of the wall. This moment is equal to

the sum of the moments of the vertical forces minus the moment of the horizontal component of the earth pressure.

The first step in determining the moment of the resultant is to find the amount, direction, and point of application of the earth pressure; and the next step is to compute the weight

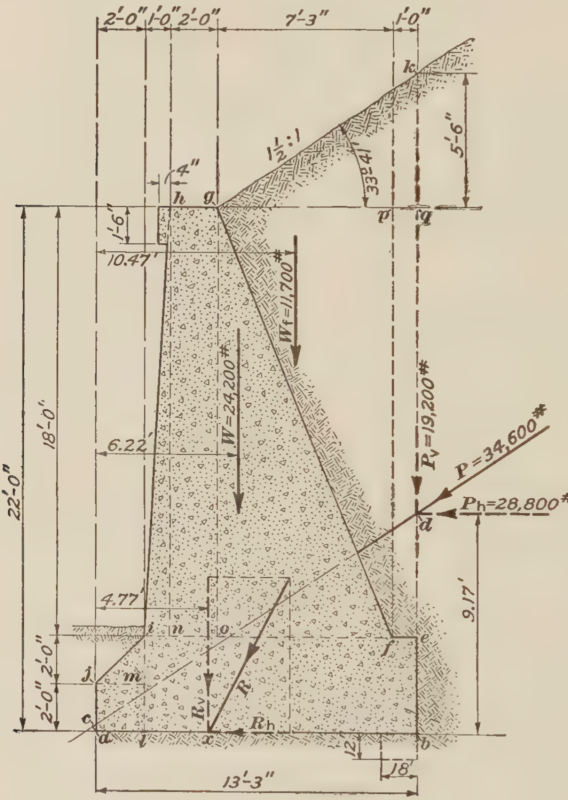


FIG. 11

of the wall, the weight of the portion of the filling that rests on the wall, and the lever arms of these weights. To determine these quantities, it is convenient to divide the sections of the wall and filling into rectangles and triangles, and to find the area in square feet of each part and its lever arm in feet about the toe of the wall. The weight of the wall per foot of

length can then be found by adding the areas of the parts of which the section of the wall is composed and multiplying the total area by the weight of a cubic foot of the masonry. Also, the distance from the toe to the line of action of that weight, which passes through the center of gravity of the wall, can be computed by multiplying the area of each part by its lever arm, adding the products, and dividing the sum by the total area of the wall. The weight of the filling and the distance from the toe to its line of action can be found in a similar manner.

The moment of the weight of the wall or filling is equal to the product of the weight and the distance from the toe to its line of action.

EXAMPLE.—Compute the moment of the resultant of all the forces acting on the wall shown in Fig. 11, which retains a bank whose surface slopes at the rate of $1\frac{1}{2}$ horizontal to 1 vertical and whose weight is 110 pounds per cubic foot. The concrete in the wall weighs 150 pounds per cubic foot.

SOLUTION.—In this case, $\tan Z = \frac{2}{3}$, $Z = 33°\ 41'$, and $h_1 = 8.25 \tan Z = 5.5$ ft. Hence, by formula 5, Art. 18,

$$P = \tfrac{1}{2}w(h+h_1)^2 \cos Z = \tfrac{1}{2}\times 110 \times 27.5^2 \times .832 = 34{,}600 \text{ lb.}$$

The horizontal component $P_h = P \cos Z = 34{,}600 \times .832 = 28{,}800$ lb., and the vertical component $P_v = P \sin Z = 34{,}600 \times .555 = 19{,}200$ lb. The lever arm of P_h is $\frac{1}{3}\times 27.5 = 9.17$ ft., and that of P_v is 13.25 ft.

To determine the weight of the wall and its lever arm, the wall section is divided into the rectangles *ajml*, *beil*, and *ghno*, and the triangles *ijm*, *hin*, and *fgo*, the projection of the coping being disregarded. The area, the lever arm, and the product of the two, or the moment, for each part of the wall are as follows:

PART	AREA, IN SQ. FT.	ARM, IN FT.	MOMENT
ajml	$2\times 2 = 4$	$\frac{1}{2}\times 2 = 1$	$4\times 1 = 4$
bcil	$11.25\times 4 = 45$	$2+\frac{1}{2}\times 11.25 = 7.63$	$45\times 7.63 = 343$
ghno	$2\times 18 = 36$	$3+\frac{1}{2}\times 2 = 4$	$36\times 4 = 144$
ijm	$\frac{1}{2}\times 2\times 2 = 2$	$\frac{2}{3}\times 2 = 1.33$	$2\times 1.33 = 3$
hin	$\frac{1}{2}\times 1\times 18 = 9$	$2+\frac{2}{3}\times 1 = 2.67$	$9\times 2.67 = 24$
fgo	$\frac{1}{2}\times 7.25\times 18 = 65.3$	$5+\frac{1}{3}\times 7.25 = 7.42$	$65.3\times 7.42 = 485$
Total	161.3		1,003

The weight W of the wall per ft. of length is equal to the product of the total area of the wall section and the weight of a cu. ft. of con-

crete, or $161.3 \times 150 = 24{,}200$ lb., and the lever arm of this weight about the toe of the wall is $1{,}003 \div 161.3 = 6.22$ ft.

In a similar manner, the earth section is divided into the rectangle *efpq* and the triangles *fgp* and *gkq*. The areas, lever arms, and moments for the parts of the filling are:

PART	AREA, IN SQ. FT.	ARM, IN FT.	MOMENT
efpq	$1 \times 18 = 18$	$13.25 - \frac{1}{2} \times 1 = 12.75$	$18 \times 12.75 = 230$
fgp	$\frac{1}{2} \times 7.25 \times 18 = 65.3$	$5 + \frac{2}{3} \times 7.25 = 9.83$	$65.3 \times 9.83 = 642$
gkq	$\frac{1}{2} \times 8.25 \times 5.5 = 22.7$	$5 + \frac{2}{3} \times 8.25 = 10.5$	$22.7 \times 10.5 = 238$
Total	106.0		1,110

Thus, the weight W_f of the filling is $106 \times 110 = 11{,}700$ lb. and its lever arm is $1{,}110 \div 106 = 10.47$ ft.

The moment of the resultant force, which is equal to the sum of the moments of W, W_f, and P_v minus the moment of P_h, is $24{,}200 \times 6.22 + 11{,}700 \times 10.47 + 19{,}200 \times 13.25 - 28{,}800 \times 9.17 = 263{,}000$ ft.-lb. Ans.

25. Position of Resultant Pressure on Base.—If the resultant force R on the wall is resolved into horizontal and vertical components at the point where it cuts the base of the footing, as shown at x in Fig. 11, the moment of the horizontal component R_h about the toe is zero because R_h passes through the center of moments, and the moment of the vertical component R_v is equal to the moment of the resultant. Hence, the distance from the toe to the point where the resultant cuts the base can be found by dividing the moment of the resultant by its vertical component. This vertical component is equal to the sum of the weight of the wall, the weight of the filling above the wall, and the vertical component of the earth pressure.

EXAMPLE.—At what distance from the toe does the resultant of the forces acting on the wall in the example of the preceding article cut the base?

SOLUTION.—As found in the preceding article, the moment of the resultant is 263,000 ft.-lb. Also, $W = 24{,}200$ lb., $W_f = 11{,}700$ lb., and $P_v = 19{,}200$ lb. Hence, the vertical component of the resultant is $W + W_f + P_v = 24{,}200 + 11{,}700 + 19{,}200 = 55{,}100$ lb., and the distance from the toe a to the point x where the resultant cuts the base is $\frac{263{,}000}{55{,}100} = 4.77$ ft. Ans.

The distance from the toe a to the edge of the middle third of the base is $\frac{1}{3} \times 13.25 = 4.42$ ft. Hence, the resultant cuts the base inside of the middle third.

26. Graphic Method.—The point where the resultant of the forces acting on a retaining wall cuts the base may also be found graphically. The section of the wall and that of the filling are divided into as few parts as possible, provided the center of gravity of each part can be readily located. Thus, in Fig. 11, the wall section could be divided into the

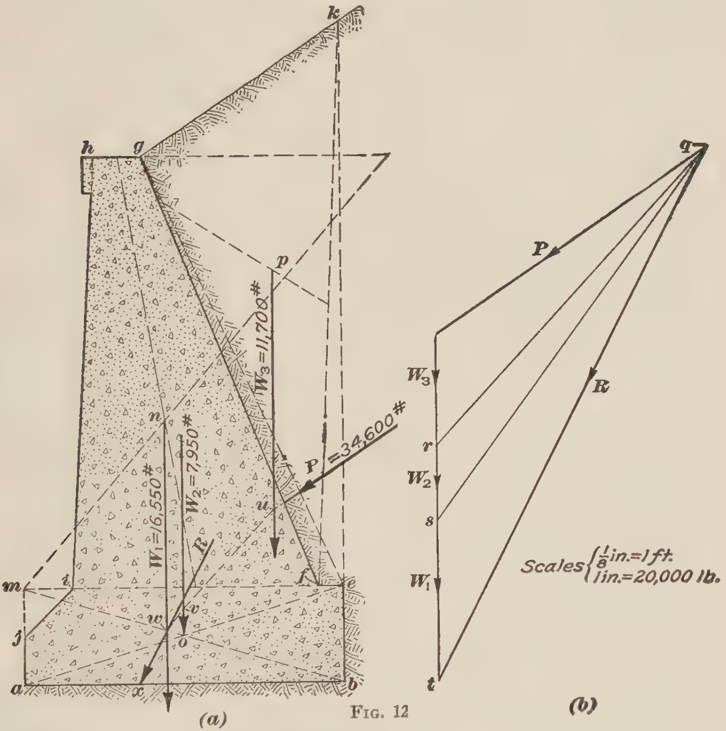


FIG. 12

trapezoids *fghi* and *ajil*, and the rectangle *beil*; and the section of the filling could be left undivided.

In order to locate graphically the resultant acting on the wall in Fig. 11, the section is redrawn in Fig. 12 (*a*) to a scale of $\frac{1}{8}$ inch = 1 foot. The work can be somewhat simplified without materially affecting the accuracy of the results by including the small triangle *ijm* as part of the footing and treating the entire footing as the rectangle *abem*. The center of gravity of each

part is located graphically, as shown by the light dashed lines. Thus, the center of gravity of *fghi* is at *n*; that of *abem* at *o*; and that of *efgk* at *p*. Next, a vertical line is drawn through the center of gravity of each part to represent the line of action of the weight of that part.

The area of the trapezoid *fghi* is $\frac{1}{2}\times(2+10.25)\times 18=110.3$ square feet and the weight W_1 of the part of the wall represented by that trapezoid is $110.3\times 150=16{,}550$ pounds. The area of the rectangle *abem* is $4\times 13.25=53$ square feet, and the weight W_2 of the corresponding part of the wall is $53\times 150=7{,}950$ pounds. The area of the quadrilateral *efgk* is $\frac{1}{2}\times(1+8.25)\times 18+\frac{1}{2}\times 8.25\times 5.5=106$ square feet, and the weight W_3 of the filling is $106\times 110=11{,}700$ pounds.

To locate the line of action of the resultant of all forces, a force polygon is constructed for the forces P, W_3, W_2, and W_1, as in (*b*); in this case, a scale of 1 inch = 20,000 pounds is used. The line *qr* represents the resultant of P and W_3, and the line of action of this resultant passes through the intersection *u* of these two forces in (*a*); hence, *uv* is drawn in (*a*) parallel to *qr* in (*b*) until it intersects the line of action of W_2 at *v*. In a similar manner, *qs* in (*b*) represents the resultant of *qr* and W_2, or the resultant of P, W_3, and W_2, and *vw* in (*a*) is drawn parallel to *qs* until it intersects the line of action of W_1 at *w*. Finally, *wx* is drawn parallel to *qt* to represent the line of action of the required resultant of all forces. The distance *ax* is then found by measurement.

EXAMPLES FOR PRACTICE

1. In example 3, page 20, the resultant earth pressure per foot of wall is 17,400 pounds and it is applied at a distance of 6.22 feet above the heel of the wall. Find (*a*) the moment of the horizontal component of the resultant earth pressure about the toe; (*b*) the amount of the vertical component of the earth pressure; and (*c*) the moment of that vertical component.

Ans. (*a*) 90,200 ft.-lb.; (*b*) 9,660 lb.; (*c*) 86,900 ft.-lb.

2. If the masonry in the wall in the preceding example weighs 150 pounds per cubic foot, calculate (*a*) the weight of the wall per foot; (*b*) the horizontal distance from the toe of the wall to the center of gravity of the wall section; (*c*) the weight of the filling resting on the

wall; and (*d*) the horizontal distance from the toe to the center of gravity of that portion of the filling.

Ans. (*a*) 11,250 lb. (*b*) 3.97 ft. (*c*) 6,250 lb. (*d*) 7.08 ft.

3. (*a*) Find the moment, about the toe, of the resultant of all the forces acting on the wall in examples 1 and 2. (*b*) Calculate the distance from the toe of the wall to the point where the resultant cuts the base.

Ans. (*a*) 85,600 ft.-lb. (*b*) 3.15 ft.

4. Find the distance in example 3 (*b*) graphically.

STABILITY AGAINST SETTLEMENT

27. Distribution of Pressure on Foundation.—The resultant of all the forces acting on a retaining wall is inclined, but only its vertical component is effective in exerting vertical pressure on the foundation. When the point where the resultant cuts the base lies within the middle third of the base, as in Fig. 13 (*a*), the entire foundation is in compression and the distribution of pressure is as indicated in (*b*). The maximum and minimum unit pressures are computed in this case by the formulas

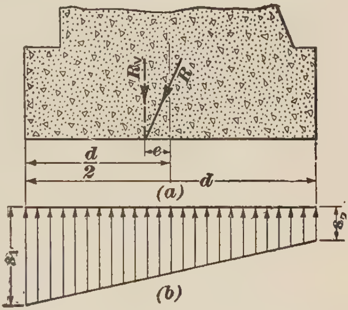


FIG. 13

$$s_1 = \frac{R_v}{d}\left(1 + \frac{6e}{d}\right) \qquad (1)$$

$$s_2 = \frac{R_v}{d}\left(1 - \frac{6e}{d}\right) \qquad (2)$$

in which s_1 = maximum unit pressure, in pounds per square foot;

s_2 = minimum unit pressure, in pounds per square foot;

R_v = vertical component of resultant pressure on foundation per foot of length of wall, in pounds;

d = width of base, in feet;

e = distance, in feet, from center of base to point of application of resultant.

If the resultant passes exactly through the center of the base, $e=0$, and

$$s_1=s_2=\frac{R_v}{d} \qquad (3)$$

When the resultant cuts the base outside of the middle third, as in Fig. 14 (a), the value of s_2 determined by formula 2 is negative, and the distribution of pressure is as indicated in (b). In this case, there is a tendency to produce tension or uplift at the edge of the base that is farther from the resultant. However, since a foundation soil cannot resist tension, it is customary to disregard the tension and to assume that the pressure varies from a maximum at the edge of the base nearer the load to zero at a distance l from that edge, as shown in (c). The distance l may be found by the formula

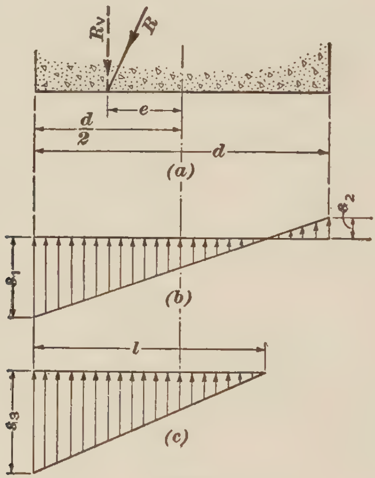


FIG. 14

$$l=1.5d-3e \qquad (4)$$

When the tension is disregarded, the maximum unit pressure s_3 is calculated by the formula

$$s_3=\frac{2R_v}{l} \qquad (5)$$

If the resultant just passes through the edge of the middle third, $l=d$.

Example 1.—What is the maximum unit pressure on the base of the wall in Fig. 11?

Solution.—Since the resultant cuts the base within the middle third, formula 1 is applied, in which R_v=55,100 lb., d=13.25 ft., and $e=\frac{13.25}{2}-4.77=1.86$ ft.

Hence,

$$s_1=\frac{R_v}{d}\left(1+\frac{6e}{d}\right)=\frac{55,100}{13.25}\times\left(1+\frac{6\times 1.86}{13.25}\right)=7,670 \text{ lb. per sq. ft.} \quad \text{Ans.}$$

Example 2.—The base of a retaining wall is 12 feet wide. If the vertical component of the resultant of all the forces acting on the wall is 40,000 pounds and it cuts the base at a distance of 3.5 feet from the toe, what is the maximum unit pressure on the foundation?

Solution.—In this case, the distance from the toe of the wall to the edge of the middle third of the base is $\frac{1}{3}\times 12=4$ ft., and the resultant cuts the base outside of the middle third. Hence, formulas 4 and 5 are applied, in which d=12 ft. and $e=\frac{12}{2}-3.5=2.5$ ft. Thus,

$$l=1.5d-3e=1.5\times 12-3\times 2.5=10.5 \text{ ft.}$$

and

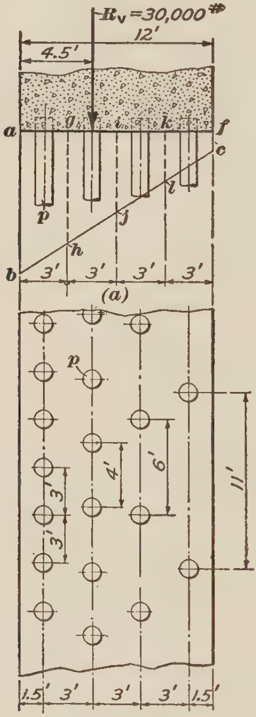
$$s_3=\frac{2R_v}{l}=\frac{2\times 40,000}{10.5}=7,620 \text{ lb. per sq. ft.} \quad \text{Ans.}$$

28. Condition of Stability Against Settlement.—When the maximum pressure on the foundation at the toe of the wall is less than the bearing strength of the soil, the wall is safe against serious settlement. If, on the other hand, the maximum pressure determined by formula 1, 3, or 5 of the preceding article exceeds the safe bearing capacity of the soil, the usual procedure is to redesign the wall, a base width being assumed that seems sufficient to reduce the soil pressure to a safe amount. For soils of low bearing capacity, however, it is often found economical to drive piles for the support of the wall.

29. Offset at Toe.—An economical and effective method of reducing the maximum soil pressure under a retaining wall is to extend the projection of the footing beyond the face of the wall. Moreover, this increases the stability of the wall against overturning. However, retaining walls are generally

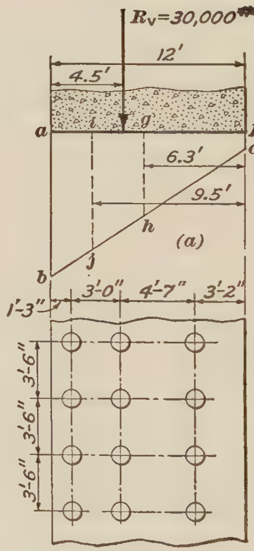
built where property is valuable or space is otherwise limited and, therefore, large toe extensions are usually prevented by lack of room.

30. Spacing of Piles.—When a retaining wall rests on piles, the bearing capacity of the foundation soil under the wall is usually neglected and it is assumed that the piles sus-



(b)

Fig. 15



(b)

Fig. 16

tain the entire load. Piles under a retaining wall are commonly arranged in rows that run either lengthwise of the wall, as in Fig. 15, or across the wall, as in Fig. 16. However, in general, the piles should not be spaced uniformly in both directions, because the pressure on the foundation varies, as shown in Figs. 13 and 14, from a maximum value at the toe of the wall to a minimum value at the heel, and it is advisable

to have the same load on each pile. The minimum spacing of the piles should be about 3 feet, as the supporting power of each pile is lowered by making the distance between them less than that amount.

When the rows run lengthwise, the first step is to divide the base of the wall into a number of strips, which are usually of equal width. Then, the total soil pressure per unit length of wall is determined for each strip, and the spacing of the piles in any strip is found by dividing the bearing capacity of a pile by the pressure per unit length in that strip.

EXAMPLE.—The footing of a retaining wall is 12 feet wide, the vertical component of the resultant force per foot length of wall is 30,000 pounds, and the point of application of the resultant is 4.5 feet from the toe of the footing. Design a pile foundation if the piles are arranged in rows that run lengthwise of the wall, and the bearing capacity of each pile is 36,000 pounds.

SOLUTION.—First, the maximum and minimum soil pressures per foot of wall are found by applying formulas 1 and 2, Art. 27, in which $R_v = 30{,}000$ lb., $d=12$ ft., and $e=\frac{12}{2}-4.5=1.5$ ft. Thus,

$$s_1=\frac{R_v}{d}\left(1+\frac{6e}{d}\right)=\frac{30{,}000}{12}\left(1+\frac{6\times 1.5}{12}\right)=4{,}375 \text{ lb. per sq. ft.}$$

and $$s_2=\frac{R_v}{d}\left(1-\frac{6e}{d}\right)=\frac{30{,}000}{12}\left(1-\frac{6\times 1.5}{12}\right)=625 \text{ lb. per sq. ft.}$$

The distribution of pressure is then represented in Fig. 15 (a) by the trapezoid $abcf$.

In this case, the width of the footing is divided into four strips, each 3 ft. wide. The unit pressures at the lines of division are

$$gh=4{,}375-\tfrac{1}{4}\times(4{,}375-625)=3{,}440 \text{ lb. per sq. ft.}$$
$$ij=4{,}375-\tfrac{1}{2}\times(4{,}375-625)=2{,}500 \text{ lb. per sq. ft.}$$
$$kl=4{,}375-\tfrac{3}{4}\times(4{,}375-625)=1{,}560 \text{ lb. per sq. ft.}$$

Hence, the total pressure per foot of wall in the left-hand strip is $\frac{ag}{2}(ab+gh)=\frac{3}{2}\times(4{,}375+3{,}440)=11{,}720$ lb.; in the next strip $\frac{gi}{2}(gh+ij)=\frac{3}{2}\times(3{,}440+2{,}500)=8{,}910$ lb.; in the third strip, $\frac{3}{2}\times(2{,}500+1{,}560)=6{,}090$ lb.; and in the right-hand strip, $\frac{3}{2}\times(1{,}560+625)=3{,}280$ lb.

The required spacing of the piles is as follows:

In the left-hand strip, $\dfrac{36{,}000}{11{,}720}=3$ ft. Ans.

In the second strip, $\dfrac{36{,}000}{8{,}910}=4$ ft. Ans.

In the third strip, $\dfrac{36{,}000}{6{,}090}=5.9$, say 6 ft. Ans.

In the right-hand strip, $\dfrac{36{,}000}{3{,}280}=11$ ft. Ans.

The piles p are therefore spaced as shown in plan in Fig. 15 (b).

Although the rows of piles should really be placed at the centers of gravity of the trapezoids in the unit-pressure diagram in (a), it is generally safe to place them at the centers of the strips, as shown in (b).

31. When the piles are arranged in rows across the wall, as in Fig. 16, the distance between rows is first assumed. Next, the required number of piles in each row is found by multiplying the vertical component of the resultant force per foot length of wall by the distance in feet between rows, and dividing the product by the safe bearing power of a pile.

The required number of piles will usually be partly decimal and the next larger whole number should be adopted. Then the spacing of the rows to develop the full capacity of the piles is computed by multiplying the bearing capacity of one pile by the number of piles in a row and dividing the result by the vertical component of the resultant force per foot of wall. Finally, the piles in each row are so arranged that each pile supports the same load. This can be accomplished by dividing the trapezoid of unit soil pressure, as $abcf$, Fig. 16 (a), into as many equal areas as there are to be piles in each row and placing a pile at the center of gravity of each division. For all practical purposes, however, the piles may be placed at the centers of these divisions. The unit pressure in each division at the edge that is farther from the heel of the wall, and the distance from the heel to that edge can be computed by the following formulas:

$$s=\sqrt{\frac{ms_1{}^2+(n-m)s_2{}^2}{n}} \qquad (1)$$

and
$$p=\frac{d(s-s_2)}{s_1-s_2} \qquad (2)$$

in which s = unit pressure, in pounds per square foot, at edge of division in question farther from heel of wall;

m = number of piles in one row between heel of wall and farther edge of division in question;

s_1 = maximum unit soil pressure at toe of wall, in pounds per square foot;

n = total number of piles in one row;

s_2 = minimum unit soil pressure at heel of wall, in pounds per square foot;

p = distance, in feet, from heel of wall to farther edge of division in question;

d = width of base of wall, in feet.

If the resultant force cuts the base of the wall outside of the middle third, the value of s_2 in the preceding formulas is zero, the distance l computed by formula 4, Art. 27, is substituted in formula 2 for the width d of the entire base, and the distances p are measured from the point of zero pressure instead of the heel of the wall. Then, formulas 1 and 2 reduce to

$$s=s_1\sqrt{\frac{m}{n}} \qquad (3)$$

$$p=\frac{s}{s_1}l \qquad (4)$$

Example.—Design a pile foundation for the conditions in the example of the preceding article, in which the piles are arranged in rows across the wall.

Solution.—Since the unit pressures at the toe and heel of the wall are, respectively, s_1=4,375 lb. per sq. ft. and s_2=625 lb. per sq. ft., the distribution of pressure is represented by the trapezoid *abcf*, Fig. 16 (*a*). Also, the vertical component of the resultant force per foot of wall is 30,000 lb., and the bearing capacity of a pile is 36,000 lb. If the distance between rows of piles is taken as 3 ft., the number of piles required in each row is $\frac{30{,}000\times 3}{36{,}000}=2.5$. Hence, 3 piles will be used in each row and, in order to develop the full bearing capacity of the piles, the distance

between rows will be made equal to $\frac{3\times 36{,}000}{30{,}000}=3.6$ ft., say 3 ft. 6 in.

The unit pressure at the edge gh of the division $cfgh$ of the pressure diagram is found by formula 1. Since $n=3$ and $m=1$,

$$s=\sqrt{\frac{ms_1^2+(n-m)s_2^2}{n}}=\sqrt{\frac{1\times 4{,}375^2+(3-1)\times 625^2}{3}}=2{,}580 \text{ lb. per sq. ft.}$$

and the distance fg can now be computed by formula 2. Thus,

$$p=\frac{d(s-s_2)}{s_1-s_2}=\frac{12\times(2{,}580-625)}{4{,}375-625}=6.3 \text{ ft.}$$

For the edge ij of the division $ghij$, $m=2$. Hence,

$$s=\sqrt{\frac{2\times 4{,}375^2+(3-2)\times 625^2}{3}}=3{,}590 \text{ lb. per sq. ft.}$$

and the distance fi is

$$p=\frac{12\times(3{,}590-625)}{4{,}375-625}=9.5 \text{ ft.}$$

The distance from the heel of the footing to the right-hand pile is $\frac{6.3}{2}$ $=3.15$ ft.$=3$ ft. 2 in.; to the center pile, $\frac{9.5+6.3}{2}=7.9$ ft.$=7$ ft. 11 in.; and to the left-hand pile, $\frac{12+9.5}{2}=10.75$ ft.$=10$ ft. 9 in. As the distance between the center and left-hand piles is a little less than the minimum allowable value of 3 ft., the middle pile will be placed 7 ft. 9 in. from the heel of the footing. In this case, the change from the computed distance is permissible because the adopted spacing of rows is slightly less than the required value. Otherwise, it would have been necessary to decrease slightly the distance between rows, so that the total load on the left-hand pile would not exceed 36,000 lb. The spacing of the piles is as shown in plan in (b).

EXAMPLES FOR PRACTICE

1. Calculate the maximum unit pressure on the foundation of a retaining wall whose base is 9 feet wide, if the vertical component of the resultant force acting on the wall is 27,200 pounds and the distance from the toe of the wall to the point where the resultant cuts the base is 3.15 feet. Ans. 5,740 lb. per sq. ft.

2. A pile foundation is to be designed for a retaining wall whose footing is 14 feet wide. The vertical component of the resultant force per foot length of wall, which is 56.000 pounds, is applied 6.5 feet from the

toe of the footing, and the bearing capacity of each pile is 50,000 pounds. If the width of the footing is divided into four strips, each 3.5 feet wide, and the piles are arranged in rows running lengthwise of the wall, what is the required spacing in the row nearest the toe of the wall?

Ans. 3.08, say 3 ft.

3. If the piles in example 2 are arranged in rows across the wall, and the number of piles in each row is taken as four, (*a*) what should be the distance between rows? (*b*) What is the distance from the heel of the wall to the pile nearest the toe?

Ans. (*a*) 3.57 ft., say 3 ft. 6 in.
(*b*) 12 ft. 6 in.

RESISTANCE TO SLIDING AND SHEAR

32. Resistance to Sliding.—The force that tends to slide a retaining wall on the foundation soil is the horizontal component of the earth pressure. The minimum resistance to sliding on the foundation is equal to the product of the vertical component of the resultant of all forces acting on the wall and the coefficient of friction of the masonry on the soil. In Table II are given approximate values of the coefficient of

TABLE II

FRICTION OF A MASONRY WALL ON VARIOUS SUBSOILS

Materials	Coefficient of Friction
Masonry on moist clay	.33
Masonry on sand	.40
Masonry on dry clay	.50
Masonry on gravel	.60
Masonry on rock, or on masonry	.65

friction for masonry on various kinds of foundations. From this table, by interpolating when necessary, the designer may select a value that will best fit any particular case, when it is desired to compute the resistance of a retaining wall to sliding.

In order to prevent sliding, the frictional resistance should exceed the horizontal component of the earth pressure, a factor of safety of about $1\frac{1}{2}$ being generally desirable. If, however, the frictional resistance is less than, or about the same as, the force tending to cause sliding, additional resistance to

sliding should be provided in one of the following ways: (1) The wall may be widened to increase its weight; (2) one or two narrow and shallow trenches may be dug in the foundation which, when filled with concrete, form keys, as *a* and *b* in Fig. 17 (*a*); (3) the base of the footing may be sloped down from the toe, as in (*b*).

In the wall shown in Fig. 11, the horizontal component of the earth pressure is 28,800 pounds and the vertical component of the resultant of all forces acting on the wall is 55,100 pounds. If the wall is to rest on a foundation of gravel, the coefficient of friction is .6, and the frictional resistance is

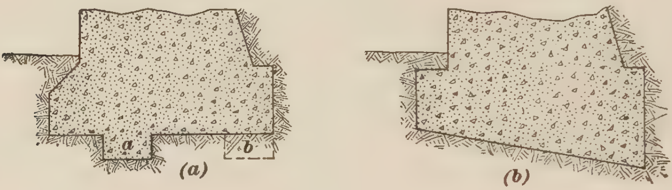


FIG. 17

$55{,}100\times .6=33{,}060$ pounds, for which the factor of safety is but $\frac{33{,}060}{28{,}800}=1.15$. Hence, additional resistance to sliding should be provided.

For a factor of safety of 1.5, the resistance to sliding should be $1.5\times 28{,}800=43{,}200$ pounds. Hence, the additional resistance required is $43{,}200-33{,}100=10{,}100$ pounds. Since the safe load on gravel is 6 tons or 12,000 pounds per square foot, the resistance offered by the soil in front of a key 12 inches high, such as is shown dotted at the heel of the wall, is ample. If the safe shearing strength of the concrete is taken as 50 pounds per square inch, or $50\times 144=7{,}200$ pounds per square foot, the required width of the key is $10{,}100\div 7{,}200=1.4$ feet, say 18 inches.

33. Resistance to Shear.—There is also a tendency for a retaining wall to slide or shear along any horizontal plane between the top and the base, because of the horizontal component of the earth pressure above that plane. Therefore, a

complete investigation of a wall with a stepped back, as in Fig. 1, would require the computation of the shearing unit stress at the level of each step; and in a wall with a sloped back, the shearing unit stress should be investigated at the level where there is an abrupt decrease in width. However, in all properly designed gravity walls, the shearing unit stress is fairly low, and in the usual types of such walls it is not necessary to investigate the shearing unit stress.

EXAMPLE.—What is the shearing unit stress just above the top of the footing in the wall in Fig. 11, if the weight of the filling is 110 pounds per cubic foot?

SOLUTION.—The first step is to compute the pressure of the earth above the plane through *if* by applying formula 5, Art. 18. In this case, $w=110$ lb. per cu. ft., $h=18$ ft., $h_1=\frac{2}{3}\times 7.25=4.83$ ft., and $Z=33°\ 41'$. Hence,

$$P=\tfrac{1}{2}w(h+h_1)^2\cos Z=\tfrac{1}{2}\times 110\times 22.83^2\times .832=23{,}900 \text{ lb.}$$

The horizontal component of that pressure is $P \cos Z=23{,}900\times .832=19{,}900$ lb. As the area at *if* for a 1-ft. length of wall is 10.25 sq. ft. or 1,476 sq. in., the shearing unit stress is $\frac{19{,}900}{1{,}476}=13$ lb. per sq. in. Ans.

Since the allowable shearing unit stress for 2,000-lb. concrete is 40 lb. per sq. in., the shearing unit stress is well below the allowable value.

EXAMPLES FOR PRACTICE

1. A retaining wall rests on a foundation of sand. If the vertical component of the resultant of all the forces acting on the wall is 27,200 pounds, what is the frictional resistance that tends to prevent the wall from sliding on its foundation? Ans. 10,900 lb.

2. A wall 12 feet high above the top of its footing retains a level fill whose weight is 100 pounds per cubic foot and whose slope of repose is $1\frac{1}{2}$ horizontal to 1 vertical. What is the shearing unit stress in the wall just above the footing, if it is 5 feet wide at that section? Ans. 3 lb. per sq. in., nearly

METHODS OF DESIGN

TYPES OF RETAINING WALLS

34. Gravity Walls.—Retaining walls constructed of stone masonry or plain concrete cannot withstand large tensile stresses and, therefore, are designed to resist overturning principally by their weight. Such walls are called gravity

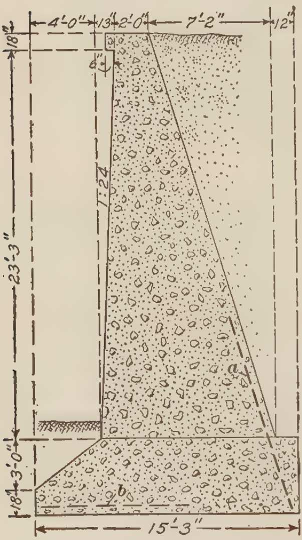


Fig. 18

Fig. 19

walls. Various cross-sections are employed for gravity retaining walls. Generally, they have a narrow top and a wide base. In nearly all designs, the face of the wall slopes slightly and the back is either sloped or stepped, as in Fig. 1. A saving in material can be effected in most cases by the use of a footing.

Concrete gravity walls are usually without steel reinforcement. However, it is sometimes found economical to add

light reinforcement to a gravity wall, as in the wall shown in Fig. 18, where the rods *a* help tie the back of the wall to the footing, and the rods *b* prevent the toe extension of the footing from breaking off.

35. Reinforced-Concrete Walls.—In a reinforced-concrete retaining wall the essential parts are a thin vertical slab, or stem, and a wide horizontal slab, or base, which are securely

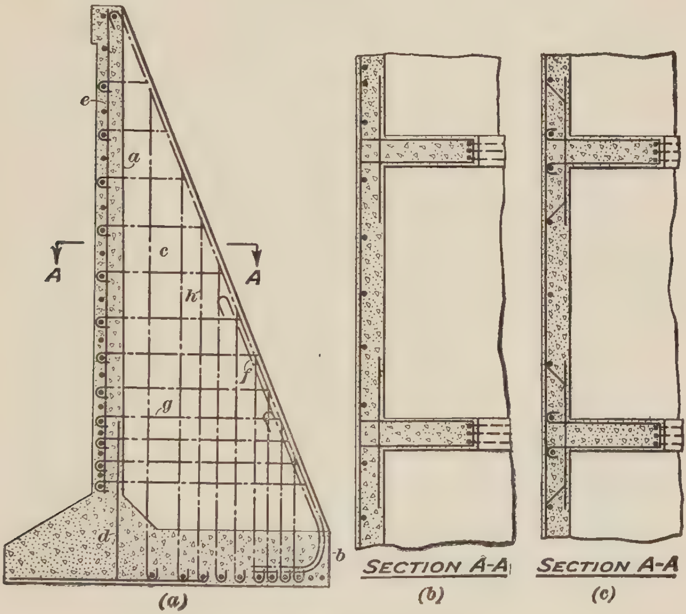


FIG. 20

tied together. By this construction, the weight of the filling behind the wall is utilized to oppose its own overturning pressure, and the amount of concrete in the wall is greatly reduced.

There are two principal types of reinforced-concrete retaining walls, namely, *cantilever walls* and *counterfort walls*. In a cantilever wall, Fig. 19, the base *a* and the stem *b* are tied together only by extending the reinforcement of the stem into the base. In a counterfort wall, Fig. 20, the stem *a* and the base *b* are joined at intervals by the brackets *c*, called counterforts, all

parts being well reinforced. In either the cantilever or counterfort type, the stem is sometimes placed at the toe of the base, the projection at the front being omitted, as in Fig. 21.

Under special conditions, other types of reinforced-concrete walls may be advantageous. For example, where the soil is soft and it is not desirable to drive piles, the maximum pressure on the foundation can be reduced by constructing a cellular retaining wall. As shown in cross-section in Fig. 22, this type

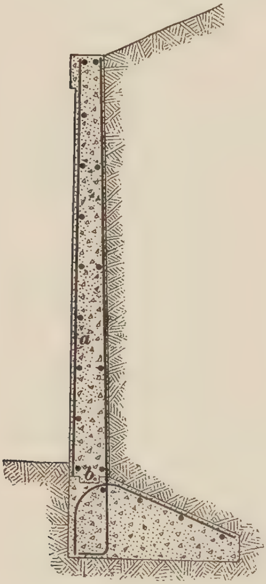


FIG. 21

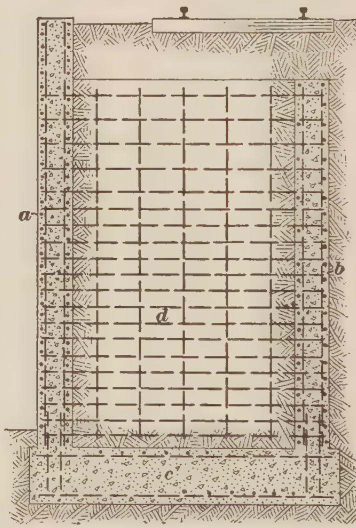


FIG. 22

of wall consists of two separate longitudinal walls *a* and *b*, resting on a base slab *c* and connected at intervals by tie walls *d*.

36. General Considerations in Design.—In selecting the most suitable type of retaining wall for a particular set of conditions, the main consideration is economy. In order to determine the section that will give the cheapest wall, it may be necessary to make several trial designs of different types of walls and to estimate their costs. Of course, considerations other than economy may also enter. For instance, a

gravity retaining wall may sometimes be most desirable because of the greater rapidity and ease with which it can be constructed as compared with a reinforced wall. It is evident that no definite rules can be given for determining offhand when a certain type of wall is most economical. In general, it will be found that for average conditions gravity retaining walls without reinforcement are most economical for heights up to 12 feet, and those with light reinforcement may often prove desirable for heights up to 15 feet. Reinforced-concrete retaining walls of the cantilever type are usually economical for heights from 12 to 20 feet, and those of the counterfort type for heights of over 18 to 20 feet.

DESIGN OF GRAVITY WALLS

SELECTION OF CROSS-SECTION

37. Outline of Method.—There are so many elements involved in the design of a retaining wall, and their combined effects are so complicated, that it is difficult to derive an equation for computing the proper wall thickness at various heights. The usual method in the case of a gravity wall is to assume a cross-section based on past experience under similar conditions and to investigate its stability against the various kinds of failure. The tentative design can then be modified, if desired, either to increase the stability of the wall or to economize on the materials.

38. General Dimensions.—In designing a gravity wall of a given height, the important dimension to establish is the width of the base. A study and comparison of many designs of existing gravity walls show that the ratio of the base width to the height of the wall is approximately constant. It varies from about .4 to about .45 for a horizontal surcharge and from .6 to .7 for a surcharge sloping at the angle of repose and is about .65 for a surcharge due to the full load on a railroad track. The width of the wall at the top of the footing should be in nearly the same ratio to the height above that level as the base width is to the total height. A satisfactory section is obtained if the heel projection does not exceed

1 foot and the toe projection is not greater than half the thickness of the footing. The top width, not including the projection of the coping, is generally between 2 and 3 feet, the limits being $1\frac{1}{2}$ and 4 feet.

The change in thickness of the wall may be obtained either by battering the face or by battering or stepping the back. A wall with a battered face generally requires a narrower base than one with a battered or stepped back; on the other hand, a wall in which the face is battered excessively is more exposed to the destructive effect of rain, does not present such a good appearance, and requires more land. Therefore, it is customary to offset the wall mainly at the back. The face is usually given a slight batter of $\frac{1}{2}$ inch to $1\frac{1}{2}$ inches per foot to improve the appearance of the wall. When the back is stepped, the limits of the masonry should lie entirely outside of the straight line joining the heel of the wall and the upper edge of the face, as ij in Fig. 1. Then, it is only necessary to investigate the shearing in any wall at the top of the footing where the wall thickness changes abruptly. However, the shearing unit stress in a gravity wall is generally so small that it can be disregarded.

The toe extension of the footing may be designed as a cantilever beam, in which case it must have sufficient strength to resist the bending moment and shear due to the maximum soil pressure on it. The weight of earth in front of the wall and resting on the toe projection is disregarded, and often the weight of the masonry in the projection is also ignored. For a plain-concrete retaining wall subjected to a maximum soil pressure of not more than 4 tons per square foot, sufficient strength is provided when the thickness of the projection is made equal to twice its length. The offset may be made in a series of steps or the top surface of the projection may be sloped, the thickness at the edge being at least 1 foot. When there is a heel projection, its thickness is generally the same as that of the toe extension.

39. Procedure in Design.—In designing a gravity retaining wall, it is first necessary to assume a tentative cross-

section for investigation. The second step is to compute the earth pressure on the back of the wall. Then the point where the resultant of all forces acting on the wall cuts the base is

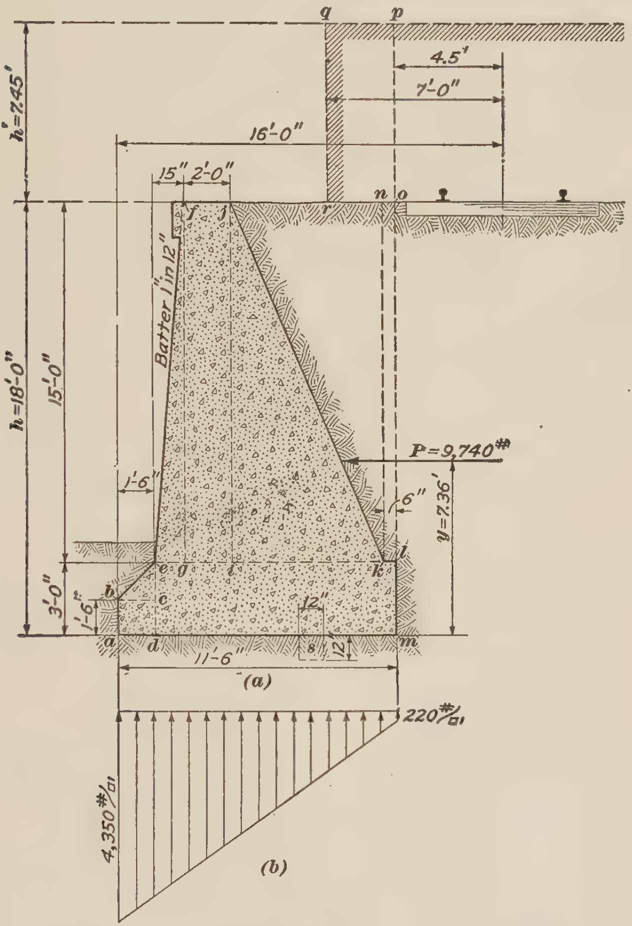


Fig. 23

located, and the stability against overturning is determined. The maximum soil pressure is next computed and compared with the allowable value. Finally, the stability against sliding is investigated.

Example.—A concrete gravity wall 14 feet high above the original ground level is to retain a level bank of earth carrying a single railroad track. The slope of repose of the filling is $1\frac{1}{2}$ horizontal to 1 vertical and its weight is 115 pounds per cubic foot. The horizontal distance from the center of the track to the toe of the wall is 16 feet and the load on the track is 12,000 pounds per foot of length. The foundation consists of fine and dry sand. Design the cross-section of the wall.

Solution.—*Tentative Cross-Section:* In order to have the foundation below the frost line, the footing will extend 4 ft. below the ground surface. Hence, the total height of the wall will be $14+4=18$ ft. and, since $.65\times 18=11.7$ ft., the trial width of the base will be made 11 ft. 6 in., as in Fig. 23 (a). The top width of the wall, exclusive of the projection of the coping which is disregarded in the computations, is taken as 2 ft.; the thickness of the footing as 3 ft.; the length of the toe projection as one-half of that thickness, or 1 ft. 6 in.; the length of the heel projection as 6 in.; and the batter of the face as 1 in. per ft.

Calculation of Earth Pressure: The horizontal distance from the heel of the wall to the center of the track is $16-11.5=4.5$ ft. Since this is less than $\frac{1}{2}\times 14=7$ ft., the loaded width of the track extends beyond the heel of the wall, and the entire load on the track is effective in producing pressure on the wall. Hence, the loaded surcharge is $\frac{12,000}{14}=857$ lb. per sq. ft. In formula 3, Art. 19, $w=115$ lb. per cu. ft., $h'=\frac{857}{115}=7.45$ ft., $h=18$ ft., and $Z=33°\,41'$. Then,

$$P=\tfrac{1}{2}wh(h+2h')\frac{1-\sin Z}{1+\sin Z}=\tfrac{1}{2}\times 115\times 18\times(18+2\times 7.45)\times\frac{1-.555}{1+.555}=9{,}740 \text{ lb.}$$

Also, by formula 4, Art. 19,

$$y=\frac{h^2+3h'h}{3(h+2h')}=\frac{18^2+3\times 7.45\times 18}{3\times(18+2\times 7.45)}=7.36 \text{ ft.}$$

Location of Resultant: The moment of the earth pressure about the toe of the wall, per ft. of length, is $9{,}740\times 7.36=71{,}700$ ft.-lb.

The calculations for the areas, arms, and moments of the parts of the wall section are as follows:

Part	Area, in sq. ft.	Arm, in ft.	Moment
abcd	$1.5\times 1.5=2.25$	$\frac{1}{2}\times 1.5=.75$	$2.25\times .75=2$
bce	$\frac{1}{2}\times 1.5\times 1.5=1.13$	$\frac{2}{3}\times 1.5=1$	$1.13\times 1=1$
efg	$\frac{1}{2}\times 1.25\times 15=9.38$	$1.5+\frac{2}{3}\times 1.25=2.33$	$9.38\times 2.33=22$
fgij	$2\times 15=30.00$	$1.5+1.25+\frac{1}{2}\times 2=3.75$	$30\times 3.75=113$
ijk	$\frac{1}{2}\times 6.25\times 15=46.88$	$2.75+2+\frac{1}{3}\times 6.25=6.83$	$46.88\times 6.83=320$
delm	$10\times 3=30.00$	$1.5+\frac{1}{2}\times 10=6.5$	$30\times 6.5=195$
Total	119.64		653

If the weight of the concrete is taken as 150 lb. per cu. ft., the total weight per ft. of wall is 119.6×150=17,900 lb. The arm of this weight about the toe is 653÷119.6=5.46 ft., and its moment is 17,900×5.46 =97,700 ft.-lb.

The values for the filling and the surcharge that rest on the wall are:

PART	AREA, IN SQ. FT.	ARM, IN FT.	MOMENT
jkn	½×6.25×15=46.88	2.75+2+⅔×6.25= 8.92	46.88×8.92=418
klon	.5×15= 7.50	11.5−½×.5=11.25	7.5×11.25= 84
opqr	(7−4.5)×7.45=18.63	11.5−½×2.5=10.25	18.63×10.25=191
Total	73.01		693

The weight of the filling and surcharge per ft. of wall is 73×115 =8,400 lb., its arm is 693÷73=9.49 ft., and its moment is 8,400×9.49 =79,700 ft.-lb.

The vertical component of the resultant is equal to the weight of the masonry, the filling, and the surcharge, which is 17,900+8,400 =26,300 lb., and the resultant moment about the toe of the footing is 97,700+79,700−71,700=105,700 ft.-lb. Hence, the distance from the toe of the wall to the point where the resultant cuts the base is 105,700 ÷26,300=4.02 ft. The distance from the toe to the edge of the middle third of the base is ⅓×11.5=3.83 ft. Since the resultant force cuts the base only about 2 in. from the edge of the middle third, this section may be considered safe and economical.

Soil Pressure: In this case, the maximum unit pressure on the foundation is found by formula 1, Art. 27. Here R_v=26,300 lb., d=11.5 ft., and $e=\frac{11.5}{2}-4.02=1.73$ ft. Hence,

$$s_1=\frac{R_v}{d}\left(1+\frac{6e}{d}\right)=\frac{26,300}{11.5}\times\left(1+\frac{6\times1.73}{11.5}\right)=4,350 \text{ lb. per sq. ft.}$$

which is less than the allowable value of 6,000 lb. per sq. ft. for fine, dry sand.

The minimum pressure at the heel is, by formula 2, Art. 27,

$$s_2=\frac{R_v}{d}\left(1-\frac{6e}{d}\right)=\frac{26,300}{11.5}\times\left(1-\frac{6\times1.73}{11.5}\right)=220 \text{ lb. per sq. ft.}$$

and the distribution of unit pressure is as represented in Fig. 23 (*b*).

Stability Against Sliding and Shearing: The horizontal force tending to cause sliding of the entire wall is 9,740 lb., and the vertical component of the resultant force is 26,300 lb. Since the coefficient of friction for masonry on sand may be taken as .4, the force resisting sliding is 26,300 ×.4=10,520 lb., which is but slightly in excess of the sliding force. If allowance is made for the additional resistance furnished by the earth

in front of the wall, no other provision against sliding is necessary. On the other hand, if the effect of the earth in front of the wall is neglected, it is advisable to introduce a key shown dotted in Fig. 23 (*a*) at *s*.

The total shear just above the top of the footing, which is equal to the resultant earth pressure at that level, is $\frac{1}{2}\times 115\times 15\times(15+2\times 7.45)\times\frac{1-.555}{1+.555}=7{,}380$ lb. The area resisting this shear is 9.5 sq. ft. or 1,368 sq. in.; and the shearing unit stress is $\frac{7{,}380}{1{,}368}=5.4$ lb. per sq. in., which is much less than the allowable 40 lb. for 2,000-lb. concrete.

EXAMPLE FOR PRACTICE

If, for the conditions in the example of the preceding article, the base width is assumed to be 11 feet, the projection of the footing at the heel of the wall is omitted, and all other dimensions of the wall are the same as in Fig. 23 (*a*), find (*a*) the vertical component of the resultant force acting on the wall, (*b*) the distance from the toe to the point where the resultant cuts the base, and (*c*) the maximum unit soil pressure.

$$\text{Ans.}\begin{cases}(a)\ 24{,}800\text{ lb.}\\(b)\ 3.58\text{ ft.}\\(c)\ 4{,}620\text{ lb. per sq. ft.}\end{cases}$$

SPECIFICATIONS FOR MATERIALS

40. Materials Used.—Gravity retaining walls are generally constructed of concrete. Stone masonry is also used to some extent, and for small walls brick is sometimes employed.

41. Concrete.—Concrete is an excellent material for the construction of retaining walls, and its use is extending rapidly. Only good material should be employed, and care should be taken to get a compact structure with a smooth face. The surface may be improved by greasing or soaping the inside of the forms and by using a richer mixture than is necessary for the interior of the wall. This can be accomplished by running the flat blade of a spade between the form and the concrete and working the handle of the spade back and forth so that more cement will flow against the form.

It is economical in the construction of retaining walls to embed large stones in the concrete when suitable stones are available. However, care should be taken to deposit these stones in such a way that they will not come too close to each other or within 3 inches of the form.

42. Walls With Cut-Stone Face.—The more common specifications for stone masonry for retaining walls call for either cut stone or rubble laid in mortar. Dry rubble is sometimes used, however, and may be quite durable.

All stone should be hard and durable, free from seams and other flaws, large and well-proportioned. When cut-stone masonry is used, the blocks should be laid on their natural beds in horizontal courses not less than 14 inches nor more than 30 inches thick, the thickness of the courses diminishing regularly from the bottom of the wall to the top. According to the appearance desired, the courses may be regular or broken, but there should be at least 50 square feet of face to each break. The surfaces of face stone should be dressed so that the mortar joints are not more than $\frac{1}{2}$ inch thick when the stone is laid. Exposed surfaces should be rock-faced, with the edges pitched to the true lines of the wall and the exact batter, and the face should not project more than 3 inches beyond the pitch line. Holes for stone hooks should not show on exposed surfaces.

43. The whole wall should be well bonded together. Headers should occupy at least one-fifth of the face of the wall. They should extend either entirely through the wall or not less than 4 feet into it, and should be evenly distributed in the wall. Furthermore, they should rest on, and be covered by, stretchers. Their width should be not less than 18 inches or the thickness of the course if that thickness is more than 18 inches. Headers in the face and back of the wall should interlock, when the thickness of the wall will permit. Stretchers should be at least 4 feet long and their width should be not less than $1\frac{1}{4}$ times the thickness. In all parts of the wall, joints should break by at least 12 inches.

44. The backing and the interior, or heart, of a wall with a cut-stone face may be constructed of concrete or of stone. When stone is used for the backing, the pieces should be large and well-shaped, and should be roughly bedded and jointed. At least one-half of the backing stone should be of the same size and character as the face stone and should have parallel

ends. The bed joints in the back of the wall should not exceed 1 inch, the vertical joints in the back should not exceed 2 inches, and the interior vertical joints should not exceed 6 inches. All stone in the heart of the wall should conform in size to the dimensions specified for the face, except as modified by necessity where less room is left between the face stone and the back stones. No space wider than 6 inches should be left to be filled with chips, or spalls, which should be laid in mortar; no spalls should be allowed in the bed joints. The coping should consist of long pieces extending the full width of the course and should have as few transverse joints as possible.

45. Rubble Masonry.—The stones for retaining walls of rubble masonry should be roughly squared and laid in irregular courses. The beds should be parallel and roughly dressed, and should be laid in a horizontal position in the wall. Face joints should not be more than 1 inch thick. At least one-fifth of the surface of the face and back of the wall should consist of headers arranged to interlock where possible. All voids in the heart of the wall should be thoroughly filled with concrete or with suitable stones and spalls fully bedded in cement mortar.

DETAILS OF CONSTRUCTION

46. Depth of Foundation.—A good foundation is essential for a retaining wall. Therefore, before a wall is built, a thorough examination of the character of the soil should be made.

Regardless of the quality of the foundation soil, footings of retaining walls generally have to be carried a few feet below the ground surface in order to avoid the dangers of frost and scour. In the northern part of the United States, 4 feet is a common minimum requirement. The earth in front of the wall above the bottom of the masonry is also of assistance in keeping the wall from sliding.

47. Coping.—The coping of a retaining wall should be so proportioned as to give the best appearance to the wall. Some

designers make the depth of the coping, in inches, equal to the exposed height of the wall, in feet. However, the depth should be not less than 12 inches and not more than about 24 inches. The top of the wall is usually horizontal, as in Fig. 1, but sometimes it is sloped forwards, as in Fig. 24, to cause the water running off the fill to drop clear of the wall. A drip *b* is often provided in the under side of the projection of the coping *a* so that the water will fall off and not trickle down the face of the wall.

48. **Drainage.**—Since a wet filling has a smaller angle of repose and weighs more than dry material, water in the filling behind a retaining wall increases the pressure on the wall. In order to provide for the escape of the water from the fill, weep holes *d*, Fig. 24, which consist of 3-inch or 4-inch drain tiles, are placed in the wall. These openings are from 10 to 25 feet apart and are placed just above the original ground to drain as much of the fill as possible. The water may be collected and brought to the weep holes by a layer *e* of broken stone, 6 to 12 inches thick, deposited along the back of the wall above the holes.

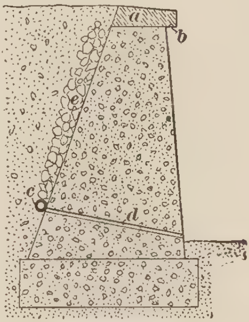


FIG. 24

If it is considered unnecessary to carry the stone to the top of the wall, it is sufficient to place a layer around the weep hole to prevent the accumulation of silt, which may block the opening. When it is expected that the quantity of water to be drained will be large, drain tiles *c*, at least 4 inches in diameter, are laid along the wall to bring the water to the weep holes.

49. **Expansion Joints.**—A monolithic concrete wall is liable to crack on account of stresses due to temperature changes and to shrinkage of the concrete while setting. Therefore, in plain-concrete retaining walls, vertical expansion joints extending across the entire wall from top to bottom are provided at regular intervals, usually from 25 to 60 feet apart.

The wall is generally constructed in alternate blocks, each as long as the distance between expansion joints. After the concrete of these blocks has hardened, the adjoining portions of the wall are poured, and the new concrete can easily be prevented from adhering to the old. Also, the surface of separation is coated with tar or an asphaltic compound, or is covered with tar paper. To hold the parts of the wall in line and to prevent seepage of water through the joints, a tongue and groove arrangement is employed. As a further protection against seepage, several layers of waterproofing material may be placed over the joints in the back of the wall; this covering should extend about $1\frac{1}{2}$ feet on each side of the joint and from the level of the weep holes to the top of the coping.

50. Waterproofing.—Water freezing in the pores of concrete may injure the structure seriously. As the concrete in a retaining wall is not usually water-tight, two coats of coal-tar pitch should be applied to the back of the wall as soon as the concrete has become sufficiently hard to permit the tar to adhere to it.

51. Depositing Fill.—Great care should be taken in depositing the fill behind a retaining wall, in order to avoid damaging the waterproofing material and to prevent excessive pressure on the back of the wall. Large boulders should not be allowed to roll against the wall. Where the fill is a mixture of rock and earth, and there is no layer of stone against the back of the wall to aid drainage, a layer of soft material should first be placed next to the waterproofing to protect it from the rock. The filling should be placed in thin, horizontal, well-compacted layers.

DESIGN OF REINFORCED-CONCRETE WALLS

GENERAL FEATURES

52. Width of Base.—In designing a reinforced-concrete retaining wall of either the cantilever or the counterfort type, it is first necessary to determine the width of base that will insure stability of the wall against overturning. As in the case of a gravity wall, the width of base should be such that the resultant of the forces acting on the wall cuts the base within the middle third. The design may be started by assuming the ratio of the width of base to the height of wall as about .45 for a horizontal surcharge, about .6 for a surcharge sloping at the angle of repose, and about .65 for a horizontal surcharge carrying the full load of a railroad track. The next step is to assume the stem and base of the wall to be rectangular slabs, each of a certain thickness, and to investigate the safety of the wall against overturning by finding the point where the resultant cuts the base. If that point is outside of the middle third, the width of the base should be increased; on the other hand, if the distance of that point from the toe of the wall is considerably more than one-third of the width of the base, economy in design could obviously be effected by decreasing the base width. The assumed thicknesses for the stem and base are merely for the purpose of investigation and may differ materially from those finally established, but a revision of the investigation will seldom be found necessary, because the effect of replacing a small amount of filling by masonry, or vice versa, is slight.

After the base width has been established to resist overturning, the wall should be investigated for stability against sliding. The minimum force resisting sliding, which is equal to the product of the vertical component of the resultant and the coefficient of friction of the wall on the foundation, is divided by the force tending to cause sliding, or the horizontal component of the earth pressure. If the quotient is greater than about $1\frac{1}{2}$, the assumed width of the base is satisfactory. If the quotient is less than $1\frac{1}{2}$, additional resistance to sliding

should be provided either by means of keys in the foundation, or by using a wider base in order to increase the weight on the wall.

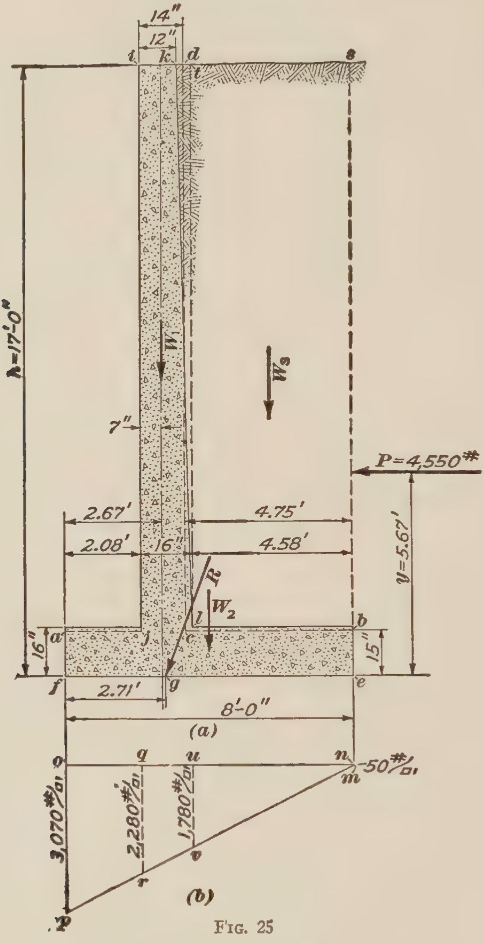


FIG. 25

EXAMPLE.—A reinforced-concrete retaining wall is to hold back a level fill of a material which weighs 110 pounds per cubic foot and has a slope of repose of 1½ horizontal to 1 vertical. If the total height of the wall is 17 feet and the center line of the stem is to be at the edge of the middle third of the base, what width of base should be adopted to resist overturning?

Solution.—If the ratio of the width of base to the height of wall is assumed to be .45, the required width of base is $.45\times17=7.65$, say 8 ft. In order that this width may be investigated before the design is started, it is further assumed that the stem will have an average thickness of 14 in. and the base will have a uniform thickness of 15 in. In the tentative section shown in Fig. 25 (*a*), the top of the base is represented by the dashed line *ab* and the back of the stem by the dashed line *cd*.

By formula 4, Art. 17, the earth pressure on the entire wall is

$$P=.143wh^2=.143\times110\times17^2=4{,}550 \text{ lb.}$$

The distance to the line of action of P from the heel e is $y=\frac{1}{3}\times17=5.67$ ft. and the moment of the earth pressure about the toe f is $4{,}550\times5.67=25{,}800$ ft.-lb.

The vertical components of the forces acting on the wall, their lever arms, and their moments about the toe are as follows:

Part	Weight per ft., in lb.	Lever Arm, in ft.	Moment, in ft.-lb.
Stem	$W_1=1.17\times15.75\times150=$ 2,770	2.67	7,400
Base	$W_2=8\times1.25\times150$ = 1,500	4	6,000
Earth	$W_3=4.75\times15.75\times110=$ 8,230	$8-\frac{1}{2}\times4.75=5.62$	46,300
Total	12,500		59,700

The vertical component of the resultant is 12,500 lb. and the resultant moment of all forces acting on the wall is $59{,}700-25{,}800=33{,}900$ ft.-lb. The distance from the toe f to the point g where the resultant R cuts the base is $\frac{33{,}900}{12{,}500}=2.71$ ft. Since the distance from the toe to the edge of the middle third of the base is $\frac{1}{3}\times8=2.67$ ft., the resultant cuts the base just inside of the middle third, and the assumed base width of 8 ft. is adequate for resisting overturning.

53. Location of Stem.—The most economical wall section is generally obtained when the center of the stem is located at a distance from the toe of the base equal to one-third of the width of the base. However, the stem may be placed anywhere along the base and there is little difference in economy for various positions between points situated one-sixth and one-half of the base width from the toe.

The position of the stem is frequently determined by other considerations. For instance, when it is desirable to place the stem at the property line, the base cannot extend beyond the face of the stem and the cross-section of the wall is then made in the form of an **L**, as in Fig. 21.

54. Batter and Top Width of Stem.—The face of a reinforced-concrete wall should preferably be given a batter of at least $\frac{1}{4}$ inch per foot. A coping is generally provided in the same manner as for a gravity wall. For practical reasons, the top of the stem is usually made at least 12 inches thick, independent of the projection of the coping, but sometimes a minimum thickness of 8 inches is allowed.

55. Expansion Joints in Reinforced-Concrete Walls. There is a difference of opinion among engineers regarding the necessity of expansion joints in reinforced-concrete walls. Some engineers consider it sufficient to provide longitudinal reinforcement; other engineers consider it necessary to use, in addition to the reinforcement, expansion joints similar to those in plain-concrete walls. When expansion joints are used, they should be spaced not more than 60 feet apart, and the wall should be cast as a unit between such joints. Longitudinal reinforcement should also be placed in the base to bridge over any soft spots in the foundation bed, but the expansion joints need not be carried through the base.

56. Drainage in Reinforced-Concrete Walls.—Weep holes not less than 4 inches in diameter should be spaced not more than 10 feet apart in reinforced-concrete walls. In counterfort walls, at least one hole should be provided in each pocket between counterforts.

DESIGN OF CANTILEVER RETAINING WALLS

57. Parts of Cantilever Wall.—For purposes of design a cantilever retaining wall in which the base projects beyond the face of the stem, as in Fig. 26, may be assumed to consist of three main parts; namely, the stem *abcd*, the projection *defg* of the base at the toe, and the projection *chij* of the base at the heel. When the wall is L shaped, it is assumed to consist of two parts, the stem and the base.

58. Shear and Bending Moment in Stem.—The stem of a cantilever wall is designed as a cantilever beam fixed at the top of the base. The load on the stem, or the earth pressure exerted on it, is determined by considering the back of the

stem as the back of a retaining wall. For instance, if the triangle *ikl*, Fig. 26 (*a*), represents the pressure of the earth on the entire wall and P the total resultant earth pressure, then *jkm* represents the pressure on the back *bc* of the stem and P' the resultant pressure on that surface. Similarly, in (*b*), the triangle *ikl* indicates the earth pressure on the entire wall, P the total resultant earth pressure, *mkn* the pressure on the back of the stem, and P' the resultant pressure on the stem.

When the fill carries no superimposed load, the distance y' from the top of the base to the point of application of P' is

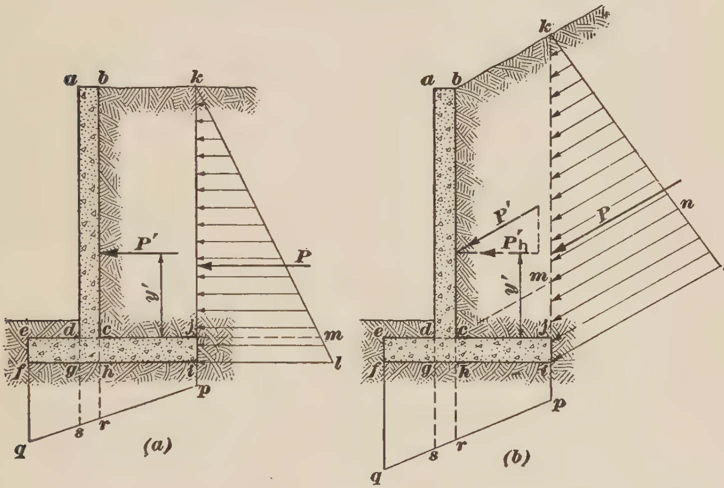


FIG. 26

equal to one-third of the height of the stem. In the case of the wall in (*a*), which has no surcharge, the maximum shear in the stem is P' and the maximum bending moment is $P'y'$. For a wall with a sloping surcharge, it is customary to consider only the horizontal component of the earth pressure in computing the shear and bending moment in the stem; thus, in the wall in (*b*), the maximum shear is P'_h and the maximum bending moment is $P'_h y'$. The compressive stress due to the weight of the wall itself may be neglected.

The shear or bending moment at any section of the stem can be found in a similar manner by considering the part of

the stem above the section as a retaining wall and computing the earth pressure on it. In walls with a level fill and no superimposed load or in walls with a sloping surcharge, the shear at any section of the stem is equal to the horizontal component of the earth pressure on the part of the stem above the section. Therefore, the shear varies as the square of the height of the stem above the section. For example, the shear at the mid-height of the stem is equal to the shear at the bottom multiplied by $(\frac{1}{2})^2$ or $\frac{1}{4}$. Also, in such walls, the bending moment at any section is equal to the product of the horizontal component of the earth pressure above the section and one-third of the height of the stem above the section. Hence, the bending moment at any section is proportional to the cube of the height of the stem above the section, so that the bending moment at the mid-height is $(\frac{1}{2})^3$ or $\frac{1}{8}$ of that at the bottom.

59. Main Reinforcement in Stem.—The main vertical reinforcing rods in the stem of a cantilever wall, as rods *c* in Fig. 19, are placed near the back of the wall, where the tensile stresses occur. Only some of these rods need extend the full height of the wall, because the required amount of reinforcement decreases toward the top of the wall. However, the maximum spacing of the rods at the top should not exceed about $2\frac{1}{2}$ times the effective thickness of the stem. When it is not desirable to use too many lengths of rods, it is customary to carry one-half of the rods half-way up the wall. In most designs, three or four different lengths are used.

Where the bending moment at any section of the stem varies with the cube of the distance below the top and the thickness of the stem is uniform for its entire height, the required steel area in the stem also varies as the cube of the height above the section.

Let h_s = height of stem, in feet;

A_s = steel area at bottom of stem, in square inches;

A_x = steel area in square inches, to be left after some rods are stopped;

h_x = theoretical distance, in feet, from top of stem to point at which steel area may be reduced to A_x.

Then,
$$\frac{A_x}{A_s}=\left(\frac{h_x}{h_s}\right)^3$$

and
$$h_x=h_s\sqrt[3]{\frac{A_x}{A_s}}$$

For instance, if half of the rods are to be stopped at a certain height, the remaining steel area $A_x=\frac{1}{2}A_s$ and the theoretical distance from the top of the stem to the point of stoppage is $h_x=h_s\sqrt[3]{.5}=.794h_s$.

When the thickness of the stem varies, but the bending moments are proportional to the cubes of the heights, the approximate distance from the top of the stem to the point where a certain number of rods may be stopped can be found by first computing h_x by the preceding formula and then assuming the next lower whole number of feet. The bending moment and the resisting moment at the assumed height should next be computed and compared. If the resisting moment is greater than the bending moment, this location of the theoretic stopping point is satisfactory. On the other hand, if the bending moment exceeds the resisting moment, a point 6 inches or a foot higher is tried next.

When the fill is loaded so that the bending moments are not proportional to the cubes of the heights, the theoretic point of stoppage for the rods must be located entirely by trial.

In all cases the rods should be carried beyond the theoretic point at which they are no longer needed, the additional length to be sufficient to develop by bond not less than one-third of their safe tensile strength and preferably one-half of that strength. The ends of the main reinforcing bars are usually anchored by being extended into the base far enough to develop by bond the maximum tensile stress in them. Another method of anchoring them is by looping them around anchor bars embedded in the base.

60. Secondary Reinforcement in Stem.—The stem should also be provided with longitudinal temperature rods, whose cross-sectional area is from .2 to .4 per cent. of the concrete area but not less than .25 square inch per foot of height. For

ease in construction, the longitudinal reinforcing rods in the stem, as rods *d* in Fig. 19, are often tied to the main vertical rods near the back. However, since the tendency for the concrete to crack is greater at the exposed surface, many engineers prefer to place from one-half to two-thirds of the longitudinal reinforcement near the face. When this latter method is adopted, the horizontal rods near the face are usually tied to light vertical spacer rods, as rods *a* in Fig. 21, which are placed at intervals of $2\frac{1}{2}$ to 3 feet. Where the longitudinal rods are spliced, they should lap a sufficient distance to develop their tensile strength by bond.

61. Design of Stem.—The stem of a cantilever wall is generally designed before the thickness of the base has been determined. Since it is necessary to know the height of the stem above the base in order to find the earth pressure on the stem, the thickness of the base is first assumed according to previous experience with similar conditions. Next the shear and bending moment at the bottom of the stem are determined. The thickness of the stem and the amount of reinforcement required there are then found by applying the principles of reinforced-concrete design. If the required thickness of the base, which is determined later, is found to differ greatly from the assumed value, the stem may have to be redesigned.

Since the bending moment and shear in the stem decrease toward the top, a saving in concrete and steel is generally effected by tapering the stem from the required thickness at the bottom to the allowable minimum value at the top and stopping some of the main vertical reinforcing rods the proper distance beyond the theoretical points where they are no longer needed to resist the tensile stress.

EXAMPLE.—If the wall in the example of Art. 52 is of the cantilever type, determine (*a*) the total maximum thickness of the stem and (*b*) the main reinforcement in it, assuming rods that can safely carry 16,000 pounds per square inch and a 2,000-pound concrete with an allowable compressive unit stress of 650 pounds per square inch.

SOLUTION.—(*a*) The cross-section of the wall that is first assumed is *dcbefaji* in Fig. 25 (*a*). For determining the earth pressure on the stem, the value of h in formula 4, Art. 17, is $17-1.25=15.75$ ft.

$$P' = .143wh^2 = .143 \times 110 \times 15.75^2 = 3{,}900 \text{ lb.}$$

and its line of action is $\frac{1}{3} \times 15.75 = 5.25$ ft. from the top of the base. The maximum shear in the stem per ft. of wall is $V = 3{,}900$ lb. and the maximum bending moment is $M' = 3{,}900 \times 5.25 = 20{,}500$ ft.-lb.

For the allowable unit stresses $f_s = 16{,}000$ and $f_c = 650$ lb. per sq. in. and $n = 15$, the constant K is 107.7. Then, the effective thickness of the stem required to resist the bending moment is

$$d = \sqrt{\frac{M'}{K}} = \sqrt{\frac{20{,}500}{107.7}} = 13.8, \text{ say } 14 \text{ in.}$$

The coefficient j is approximately .875 and the shearing unit stress at the bottom of the stem is

$$v = \frac{V}{bjd} = \frac{3{,}900}{12 \times .875 \times 14} = 26.5 \text{ lb. per sq. in.}$$

which is well below the allowable value of 40 lb. per sq. in.

If 2 in. of concrete is allowed outside the steel, the total thickness of the stem at the bottom is 16 in. Ans.

If the stem is made 12 in. thick at the top, as indicated by the full line kl in Fig. 25 (a), the average thickness is $\frac{12+16}{2} = 14$ in., as assumed.

(b) The required steel area per in. of wall is

$$A_s = \frac{M'}{f_s jd} = \frac{20{,}500}{16{,}000 \times .875 \times 14} = .1047 \text{ sq. in.}$$

which can be provided by means of $\frac{7}{8}$-in. round rods spaced $\frac{.601}{.1047} = 5\frac{3}{4}$ in. on centers, as shown in Fig. 27.

62. Design of Toe Projection.—When the wall is **L** shaped, as in Fig. 21, there is no toe projection. In **T**-shaped walls, like those shown in Figs. 19 and 27, the toe projection is designed as a cantilever fixed at the face of the stem. The only loads that are usually assumed to act on the cantilever are the weight of the projection itself and the upward soil reaction corresponding to $fgsq$ in Fig. 26. The weight of the earth carried by the projection is neglected, and often the weight of the projection is also disregarded. Generally, the thickness of the toe cantilever is determined by the shear, which is greatest at the face of the stem. When the toe cantilever is comparatively thin, it is made rectangular in cross-section, as in

Figs. 19 and 27, but in some designs its top is sloped from the face of the stem to the toe of the wall, as shown for the counterfort wall in Fig. 20 (*a*).

In the case of the toe projection, the main reinforcement is near the bottom and at least 3 inches of concrete below the center of the steel is required for protection. Often, this

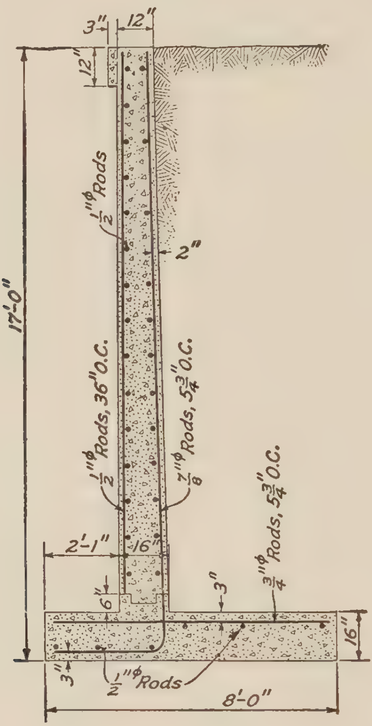


Fig. 27

reinforcement is conveniently provided by bending the rods that form the main reinforcement in the stem, as in Fig. 19, but sometimes separate rods are introduced in the toe projection. Longitudinal reinforcement is also inserted.

Example.—Determine the thickness of the toe projection and the main reinforcement in it for the wall in the example of the preceding article.

SOLUTION.—The first step is to determine the distribution of the upward soil reaction. In formulas 1 and 2, Art. 27, $R_v=12{,}500$ lb., $d=8$ ft., and $e=\frac{8}{2}-2.71=1.29$ ft. Then,

$$s_1=\frac{R_v}{d}\left(1+\frac{6e}{d}\right)=\frac{12{,}500}{8}\times\left(1+\frac{6\times1.29}{8}\right)=3{,}070 \text{ lb. per sq. ft.}$$

$$s_2=\frac{R_v}{d}\left(1-\frac{6e}{d}\right)=\frac{12{,}500}{8}\times\left(1-\frac{6\times1.29}{8}\right)=50 \text{ lb. per sq. ft.}$$

The distribution of pressure on the foundation is represented by the trapezoid *mnop* in Fig. 25 (*b*). The unit pressure at the face of the stem is $50+\frac{5.92}{8}\times(3{,}070-50)=2{,}280$ lb. per sq. ft. and the total upward pressure on the toe cantilever, which is represented by the area of the trapezoid *oprq*, is $2.08\times\left(\frac{3{,}070+2{,}280}{2}\right)=5{,}560$ lb. If the cantilever is assumed to have a uniform thickness of 1.25 ft., its weight is $2.08\times1.25\times150=390$ lb. and the shear at the edge of the stem is $5{,}560-390=5{,}170$ lb.

The approximate effective depth of the toe cantilever required to resist the shear is

$$d=\frac{V}{jbv}=\frac{5{,}170}{.875\times12\times40}=12.3, \text{ say } 13 \text{ in.}$$

and if 3 in. of concrete is allowed below the steel, the total thickness is 16 in. Ans.

The difference between the required thickness of 16 in. and the assumed value of 15 in. is so small that the value of the shear is not affected appreciably.

The moment of the soil reaction about the edge of the stem is $2.08\times2{,}280\times\frac{2.08}{2}+\frac{1}{2}\times2.08\times(3{,}070-2{,}280)\times\frac{2}{3}\times2.08=6{,}070$ ft.-lb.; the moment of the weight of the masonry is $2.08\times1.33\times150\times\frac{2.08}{2}=432$ ft.-lb.; and the bending moment at the edge of the stem is $6{,}070-430=5{,}640$ ft.-lb. The approximate area of steel per in. of wall is

$$A_s=\frac{M'}{f_sjd}=\frac{5{,}640}{16{,}000\times.875\times13}=.031 \text{ sq. in.}$$

which can be provided by means of $\frac{1}{2}$-in. round rods spaced $\frac{.196}{.031}=6.3$, say 6 in. on centers. However, in this case the reinforcement can be conveniently provided by continuing the horizontal portions of the reinforcing rods for the stem, as in Fig. 27.

63. Design of Heel Projection.—The heel projection of a cantilever wall is assumed to act as a cantilever beam fixed at the back of the stem. Where the earth pressure is horizontal, as in Fig. 26 (a), the forces acting downwards on the heel projection are the weight of the filling directly above it, the load carried by that filling if there is a superimposed load, and the weight of the projection. When the earth pressure is inclined, as in (b), it is necessary to include also its vertical component. The earth pressure per foot on the heel projection is greater at the heel than at the back of the stem. However, in designing the heel projection, it may be assumed that the vertical component of the total pressure P is uniformly distributed over the projection. The downward forces acting on the heel projection are partly balanced by the upward reaction of the soil. If the distribution of the soil pressure in the illustration is represented by the trapezoid $fipq$, the force acting upwards on the heel projection is the reaction of the soil corresponding to the area $hipr$.

The main reinforcement of the heel projection is placed near the top and is anchored by carrying the rods far enough beyond the back of the stem and sometimes also by hooking their ends, as shown in Figs. 19 and 21. A covering of 2 inches of concrete is considered sufficient by some engineers but others recommend 3 inches. Longitudinal reinforcement should also be placed under the main rods and tied to them.

The thickness of the heel projection at the back of the stem should be sufficient to resist the bending moment at that section, the maximum tension being produced there. In some cases, the maximum shear in the heel projection occurs at a section between the back of the stem and the heel of the wall, because the upward soil reaction decreases toward the heel. However, when the stem is placed at or near the forward one-third point of the base, the shear in the heel projection at the back of the stem is either a maximum or only slightly less than the maximum value, and for all practical purposes it may be assumed that the maximum shear occurs at the back of the stem. When the toe projection is short or the wall is L shaped, and especially when the heel projection is sloped

in order to effect a saving in concrete, it is necessary also to determine the shear and shearing unit stress at intermediate points between the back of the stem and the heel of the wall. Generally it is sufficient to make such investigations at intervals of 2 feet. In case the actual shearing unit stress at any section exceeds the allowable value, the thickness of the heel projection should be increased sufficiently to resist the shear, because web reinforcement in the base of a cantilever wall is undesirable.

EXAMPLE.—Determine the maximum thickness and main reinforcement for the heel projection of the cantilever wall in the examples of Arts. 61 and 62.

SOLUTION.—As shown in Fig. 25 (a), the length of the heel projection is 4.58 ft. Its thickness will be assumed the same as that required for the toe cantilever, or 16 in. The forces acting on the heel projection are the weight of the filling in the width st or 4.58 ft., the weight of the masonry in the width of 4.58 ft., and the upward soil reaction represented by the area of the trapezoid $mnuv$. The weight of the filling is $4.58\times15.67\times110=7{,}890$ lb., its lever arm about the edge l of the stem is $\frac{1}{2}\times4.58=2.29$ ft., and its moment is $7{,}890\times2.29=18{,}070$ ft.-lb. The weight of the masonry is $4.58\times1.33\times150=4.58\times200=916$ lb., its arm is 2.29 ft., and its moment is $916\times2.29=2{,}100$ ft.-lb. Since the unit soil pressure at the edge l is $50+\dfrac{4.58}{8}\times(3{,}070-50)=1{,}780$ lb. per sq. ft., the moment of the upward pressure on the entire heel projection is $4.58\times50\times\dfrac{4.58}{2}+\frac{1}{2}\times4.58\times(1{,}780-50)\times\frac{1}{3}\times4.58=6{,}570$ ft.-lb.

The bending moment at l is $18{,}070+2{,}100-6{,}570=13{,}600$ ft.-lb. and the effective depth required to resist this bending moment is

$$d=\sqrt{\frac{M'}{K}}=\sqrt{\frac{13{,}600}{107.7}}=11.2 \text{ in.}$$

However, the total thickness of the heel projection should be the same as that of the toe cantilever, or 16 in. as found in the preceding article. If the steel is placed 3 in. from the surface, the effective depth is 13 in.

The maximum shearing unit stress will now be determined. The sum of the downward forces on the entire heel cantilever per ft. of wall is $7{,}890+920=8{,}810$ lb., and the upward force is $4.58\times\left(\dfrac{1{,}780+50}{2}\right)$ $=4{,}190$ lb. Hence, the shear at the edge of the stem is $8{,}810-4{,}190$ $=4{,}620$ lb., and the corresponding shearing unit stress is

$$v=\frac{V}{bjd}=\frac{4{,}620}{12\times .875\times 13}=34 \text{ lb. per sq. in.}$$

which is less than the allowable value of 40 lb. per sq. in., and the total thickness of 16 in. is satisfactory. Ans.

The approximate area of steel per in. of wall in the heel cantilever is

$$A_s=\frac{M'}{f_s jd}=\frac{13{,}600}{16{,}000\times .875\times 13}=.0747 \text{ sq. in.}$$

which can be provided by means of ¾-in. round rods spaced $\frac{.442}{.0747}=5.92$, say 5¾ in. on centers, as in Fig. 27.

64. Construction of Cantilever Walls.—In order to provide the proper details for the connection of the stem to the base of a cantilever retaining wall, it is necessary to consider the method that is to be followed in constructing the wall. When the entire wall section is cast as a unit, there is considerable difficulty in bracing the forms for the stem and in keeping the reinforcing rods for the stem in position. Usually, the base of the wall is cast first, and a tongue-and-groove construction joint is provided between the base and stem, as at *b* in Fig. 21. Often it is found advantageous to cast a part of the stem with the base, as in Fig. 19, because the forms for the stem can then be properly spaced by the stump and held in place by bolts through it. To prevent shearing at the construction joint, the concrete first placed should be provided with either a projecting key, as in Fig. 19, or a notch, as in Fig. 21.

In order to avoid the necessity of keeping the vertical rods of the stem in position during the construction of the base, dowels may be inserted in the base and the main rods of the stem can be tied to them later. A dowel should be provided for each main rod in the stem, and it should project far enough above and below the construction joint to develop by bond the safe tensile strength of the main rod.

65. Summary of Design.—The first step in the design of a cantilever retaining wall is to assume a tentative cross-section, to determine the earth pressure on the entire wall, and to locate the point where the resultant of all the forces acting on the wall cuts the base. If the resultant falls out-

side, or too far inside, of the middle third, the base width is changed and a similar procedure is followed for the new section. When a base width that is satisfactory from the standpoint of overturning is determined in this way, the

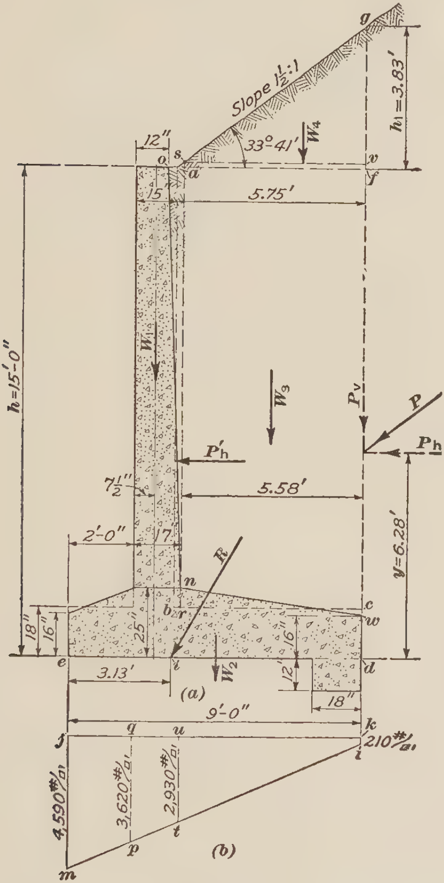


Fig. 28

stability of the wall against sliding is investigated and, if necessary, additional resistance to sliding is provided. The distribution of the soil pressure is then found, and the stem and toe and heel cantilevers are finally designed.

The toe and heel cantilevers should preferably have the same thickness at the stem. Therefore, it is advisable to determine the approximate maximum thickness required for each cantilever before the reinforcement of either projection is designed. The projection for which the approximate thickness is greater should then be designed first.

EXAMPLE.—A reinforced-concrete retaining wall of the cantilever type is to hold back an embankment whose surface slopes at the angle of repose. The total height of the wall is 15 feet, and the toe projection is limited to 2 feet. The weight of the filling is 100 pounds per cubic foot, and its slope of repose is 1½ horizontal to 1 vertical. The foundation material is gravel. Design the cross-section, assuming reinforcement that can safely carry 16,000 pounds per square inch and 2,000-pound concrete for which the allowable compressive unit stress is 800 pounds per square inch.

SOLUTION.—*Tentative Cross-Section:* The ratio of the width of base to the height will be assumed as .6, and a base width of $.6\times15=9$ ft. will be tried. In order to investigate the stability of the wall against overturning and sliding before the parts are designed, it is necessary to assume also the thickness of the stem and that of the base. For simplicity in the calculations, the stem will be treated as a rectangle 15 in. wide and the base as a rectangle 18 in. high. For the tentative section in Fig. 28, the back of the wall is represented by the dashed line ab and the top of the footing by bc.

Earth Pressure: The heel projection is $9-2-1.25=5.75$ ft., $Z=33°\ 41'$, and $h_1=5.75 \tan Z=5.75\times\frac{2}{3}=3.83$ ft. By formula 5, Art. 18,

$$P=\tfrac{1}{2}w(h+h_1)^2 \cos Z=\tfrac{1}{2}\times100\times18.83^2\times.832=14{,}750 \text{ lb.}$$

The distance to the point of application of P from the heel d is $y=\frac{1}{3}\times18.83=6.28$ ft. The horizontal component of P is $P_h=P\cos Z=14{,}750\times.832=12{,}270$ lb. and the vertical component is $P_v=P\sin Z=14{,}750\times.555=8{,}190$ lb.

Location of Resultant: The vertical components of the forces acting on the wall, their lever arms, and their moments about the toe e are as follows:

PART	WEIGHT PER FT., IN LB.		LEVER ARM, IN FT.	MOMENT, IN FT.-LB.
Stem	$W_1=1.25\times13.5\times150$	= 2,530	2.63	6,650
Base	$W_2=9\times1.5\times150$	= 2,030	4.5	9,140
abcf	$W_3=5.75\times13.5\times100$	= 7,760	$9-\frac{1}{2}\times5.75=6.12$	47,490
afg	$W_4=\frac{1}{2}\times5.75\times3.83\times100$	= 1,100	$9-\frac{1}{3}\times5.75=7.08$	7,790
P_v		= 8,190	9	73,710
Total		21,610		144,780

Hence, the vertical component of the resultant is 21,600 lb., and the moment of all vertical components is 144,800 ft.-lb. The moment of the horizontal component of the earth pressure is $12{,}270\times 6.28=77{,}100$ ft.-lb., and the moment of the resultant is $144{,}800-77{,}100=67{,}700$ ft.-lb. The distance from the toe e to the point i where the resultant R cuts the base is $\dfrac{67{,}700}{21{,}600}=3.13$ ft. Since the distance from the toe to the forward edge of the middle third is $\frac{1}{3}\times 9=3$ ft., the resultant cuts the base .13 ft. or $1\frac{1}{2}$ in. within the middle third, and the assumed width of base of 9 ft. is satisfactory to resist overturning.

Resistance to Sliding: From Table II, the coefficient of friction between the wall and its foundation of gravel is .6 and the minimum resistance to sliding is $21{,}600\times .6=12{,}960$ lb. This value is not much greater than the horizontal component of the earth pressure, which is 12,270 lb., and a key should be provided at the heel of the wall, as shown in Fig. 28 (a). The safe load on a gravel soil is 12,000 lb. per sq. ft. For a factor of safety of $1\frac{1}{2}$, the additional resistance to sliding to be offered by the soil in front of the key is $12{,}270\times 1.5-12{,}960=5{,}450$ lb., and the key shown is amply strong.

Distribution of Soil Pressure: Since the resultant of all the forces acting on the wall cuts the base within the middle third, the maximum and minimum soil pressures are found by formulas 1 and 2, Art. 27, in which $R_v=21{,}600$ lb., $d=9$ ft., and $e=\frac{9}{2}-3.13=1.37$ ft. Hence,

$$s_1=\frac{R_v}{d}\left(1+\frac{6e}{d}\right)=\frac{21{,}600}{9}\times\left(1+\frac{6\times 1.37}{9}\right)=4{,}590 \text{ lb. per sq. ft.}$$

$$s_2=\frac{R_v}{d}\left(1-\frac{6e}{d}\right)=\frac{21{,}600}{9}\times\left(1-\frac{6\times 1.37}{9}\right)=210 \text{ lb. per sq. ft.}$$

The distribution of pressure on the foundation is represented by the trapezoid $jklm$ in Fig. 28 (b).

Design of Stem: For determining the earth pressure on the stem, the thickness of the base at the stem will be assumed as 2 ft. Then, the value of h in formula 4, Art. 18, is $15-2=13$ ft., and

$$P'=\tfrac{1}{2}wh^2\cos Z=\tfrac{1}{2}\times 100\times 13^2\times .832=7{,}030 \text{ lb.}$$

The horizontal component of this earth pressure is

$$P'_h=P'\cos Z=7{,}030\times .832=5{,}850 \text{ lb.}$$

and its lever arm about the top of the base is $\frac{1}{3}\times 13=4.33$ ft. Therefore, the maximum shear in the stem per ft. of wall is $V=5{,}850$ lb. and the maximum bending moment is $M'=5{,}850\times 4.33=25{,}300$ ft.-lb.

For the allowable unit stresses $f_s=16{,}000$ and $f_c=800$ lb. per sq. in. and for $n=15$, the constant K is 147.1. Then,

$$d\doteq\sqrt{\frac{M'}{K}}=\sqrt{\frac{25{,}300}{147.1}}=13.1, \text{ say 14 in.}$$

The coefficient j is approximately .857 and the effective depth required to resist the shear is

$$d=\frac{V}{jbv}=\frac{5{,}850}{.857\times12\times40}=14.2, \text{ say } 15 \text{ in.}$$

If 2 in. of concrete is allowed outside the steel, the total thickness of the stem at the bottom is 17 in. The stem will be tapered to a width of

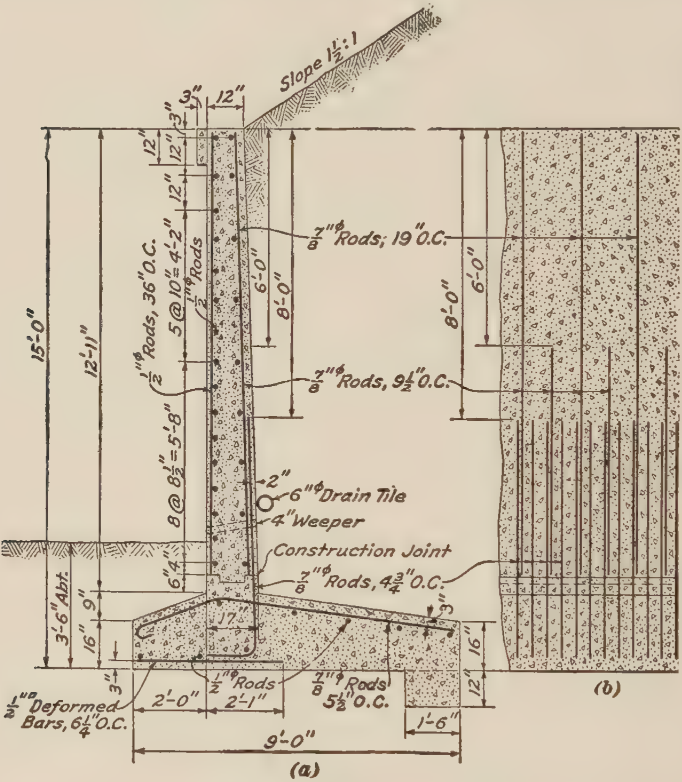


Fig. 29

12 in. at the top, as shown by the full line no in Fig. 28 (a) and in the adopted section in Fig. 29 (a).

The required steel area per in. of wall is

$$A_s=\frac{M'}{f_sjd}=\frac{25{,}300}{16{,}000\times.857\times15}=.123 \text{ sq. in.}$$

If ⅞-in. round rods are used, the required spacing is $\frac{.601}{.123}=4.89$, say 4¾ in.

Then, the number of rods per ft. of wall is $\frac{12}{4.75}=2.53$, their total perimeter is $2.53\times2.75=6.96$ in., and the unit bond stress is

$$u=\frac{V}{jdO}=\frac{5,850}{.857\times15\times6.96}=65.4 \text{ lb. per sq. in.,}$$

which is less than the allowable value of 80 lb. per sq. in. for plain bars. Hence, the $4\frac{3}{4}$ in. spacing is satisfactory at the bottom of the stem.

As shown in Fig. 29 (b), every fourth rod will be carried all the way up the wall so that the spacing of the rods at the top will be $4\times4\frac{3}{4}=19$ in. Half of the rods will be stopped about 25 diameters, or $25\times\frac{7}{8}=22$ in., say 2 ft., above the plane where the spacing may be increased to $2\times4\frac{3}{4}=9\frac{1}{2}$ in.; and the rods midway between the long rods, or the intermediate rods, will be stopped 2 ft. above the plane where the spacing may be increased to 19 in. By the formula of Art. 59, the theoretical distance from the top of the stem to the plane where half of the rods may be stopped, or where $A_x=\frac{1}{2}A_s$, is

$$h_x=h_s\sqrt[3]{\frac{A_x}{A_s}}=.794h_s=.794\times13=10.3 \text{ ft.}$$

Since the stem tapers, a distance of 10 ft. will be tried. At this section, the bending moment per ft. of wall is $\left(\frac{10}{13}\right)^3\times25,300=11,500$ ft.-lb. The effective thickness of the stem is $10+\frac{10}{13}\times(15-10)=13.8$ in., the area of steel per ft. of wall is $\frac{.601\times2.53}{2}=.76$ sq. in., $p=\frac{.76}{12\times13.8}=.00459$, $j=.897$, and the resisting moment is $A_sjdf_s=.76\times.897\times13.8\times16,000=150,500$ in.-lb. or 12,500 ft.-lb. Hence, half the rods are sufficient at a depth of 10 ft., and the short rods will be carried to a point $10-2=8$ ft. from the top of the stem.

For the intermediate rods, $A_x=\frac{1}{4}A_s$ and

$$h_x=h_s\sqrt[3]{.25}=13\times.63=8.2 \text{ ft.}$$

At a depth of 8 ft., the bending moment is $\left(\frac{8}{13}\right)^3\times25,300=5,900$ ft.-lb., the effective thickness is $10+\frac{8}{13}\times(15-10)=13.1$ in., $A_s=\frac{.601\times2.53}{4}=.38$ sq. in. per ft. of wall, $p=\frac{.38}{12\times13.1}=.00242$, $j=.922$, and the resisting moment is $.38\times.922\times13.1\times16,000=73,400$ in.-lb. or 6,120 ft.-lb. Therefore, the intermediate rods will be stopped $8-2=6$ ft. from the top of the stem.

In this case, the base will be cast first with 6 in. of the stem, as in Fig. 29 (a). Hence, the dowels should project $50\times\frac{7}{8}=44$ in. above the construction joint. However, if the dowels are carried to a point 8 ft. from the top of the stem, as shown in the adopted section, they can also serve as the main reinforcement up to that level.

The area of the longitudinal reinforcement in the stem will be taken as .2 per cent. of the total concrete area; two-thirds of the steel will be placed near the face, and the other third near the back. At the bottom of the stem, the total thickness of the concrete is 17 in. and the required amount of steel per in. of height near the face is $.002\times17\times\frac{2}{3}=.0227$ sq. in. For $\frac{1}{2}$-inch round rods, the spacing is $.196\div.0227=8.63$, say $8\frac{1}{2}$ in. At the middle of its height, the stem is $\frac{1}{2}\times(12+17)=14.5$ in. thick, and the required steel area near the face is $.002\times14.5\times\frac{2}{3}=.0193$ sq. in. per in. Above this point, the spacing of the rods may be increased to $.196\div.0193=10.15$, or 10 in. The rods near the back will be opposite the alternate rods near the face, as shown in Fig. 29 (a). The slope of the earth in back of the wall begins at point o, Fig. 28 (a), and not at the assumed point a; but, unless there is much difference between the tentative and the adopted thicknesses for the top of the stem, it may be assumed throughout the design that the slope begins at a.

Design of Base: The unit soil reaction at the face of the stem is $210+\frac{7}{9}\times(4{,}590-210)=3{,}620$ lb. per sq. ft. Then, the total pressure on the toe cantilever, per ft. of wall, which is represented by the area of the trapezoid $jmpq$, Fig. 28 (b), is $2\times\left(\frac{4{,}590+3{,}620}{2}\right)=8{,}210$ lb. If the cantilever is assumed to have an average thickness of 1.5 ft., its weight is $1.5\times2\times150=450$ lb., and the shear at the edge of the stem is $8{,}210-450=7{,}760$ lb.

The moment of the soil pressure about the edge of the stem is $2\times3{,}620\times\frac{2}{2}+\frac{1}{2}\times2\times(4{,}590-3{,}620)\times\frac{2}{3}\times2=8{,}530$ ft.-lb.; the moment of the weight of the masonry is $450\times\frac{2}{2}=450$ ft.-lb.; and the resultant moment is $8{,}530-450=8{,}080$ ft.-lb.

The approximate effective thickness of the toe cantilever at the edge of the stem required to resist the shear is

$$d=\frac{V}{jbv}=\frac{7{,}760}{.857\times12\times40}=18.9 \text{ in.}$$

and that required to resist the bending moment is

$$d=\sqrt{\frac{M'}{K}}=\sqrt{\frac{8{,}080}{147.1}}=7.4 \text{ in.}$$

The required approximate thickness of the heel cantilever will now be found.

As shown by the dashed line *cr*, Fig. 28 (*a*), the average thickness of the heel projection is first assumed to be 18 in. The forces acting downwards on the heel cantilever are the weight of the filling represented by the area *crsg*, the weight of the masonry in the heel projection, and the vertical component P_v of the earth pressure. These forces are resisted by the part of the soil reaction represented by the trapezoid *kltu* in (*b*).

In Fig. 28 (*a*), $fv=\frac{2}{3}\times(5.75-5.58)=.11$ ft., and $rs=15+.11-1.5=13.61$ ft. Hence, the weight of the filling in the rectangle *crsv* is $5.58\times13.61\times100=7{,}590$ lb.; and that in the triangle *svg* is $\frac{1}{2}\times5.58\times(3.83-.11)\times100=1{,}040$ lb. The weight of the masonry in the heel cantilever without the 12″×18″ key is $5.58\times1.5\times150=1{,}260$ lb.

At the back of the stem, the unit soil reaction is $210+\frac{5.58}{9}\times(4{,}590-210)=2{,}930$ lb. per sq. ft. Hence, the total upward pressure on the heel projection per ft. of wall, which is represented by the area of the trapezoid *kltu* in (*b*), is $5.58\times\left(\frac{2{,}930+210}{2}\right)=8{,}760$ lb. If the vertical component of the earth pressure is assumed to be uniformly distributed over the heel projection, the sum of the moments of the downward forces about *r* is $7{,}590\times\frac{5.58}{2}+1{,}040\times\frac{2}{3}\times5.58+1{,}260\times\frac{5.58}{2}+8{,}190\times\frac{5.58}{2}=51{,}400$ ft.-lb.

The moment of the upward soil reaction is $5.58\times210\times\frac{5.58}{2}+\frac{1}{2}\times5.58\times(2{,}930-210)\times\frac{5.58}{3}=17{,}400$ ft.-lb. and the bending moment at *r* is $51{,}400-17{,}400=34{,}000$ ft.-lb. The effective thickness required to resist this moment is, approximately,

$$d=\sqrt{\frac{M'}{K}}=\sqrt{\frac{34{,}000}{147.1}}=15.2, \text{ say 16 in.}$$

The shear at *r* is $7{,}590+1{,}040+1{,}260+8{,}190-8{,}760=9{,}320$ lb., and the approximate effective thickness required to resist that shear is

$$d=\frac{V}{jbv}=\frac{9{,}320}{.857\times12\times40}=22.7 \text{ in.}$$

Since the approximate maximum thickness for the heel cantilever is greater than that for the toe cantilever, the heel cantilever will be designed first.

If an effective thickness of 23 in. is assumed, the approximate area of steel per in. of width in the heel projection is

$$A_s = \frac{M'}{f_s jd} = \frac{34{,}000}{16{,}000 \times .857 \times 23} = .108 \text{ sq. in.}$$

For these values, $p = \frac{.108}{23} = .0047$ and the corresponding value of j is .896.

A more accurate value of the required effective thickness is

$$d = \frac{9{,}320}{.896 \times 12 \times 40} = 21.7, \text{ say } 22 \text{ in.}$$

Then, $A_s = \frac{34{,}000}{16{,}000 \times .896 \times 22} = .108$ sq. in. per in.

If $\frac{7}{8}$-in. round rods are used, their spacing may be $\frac{.601}{.108} = 5.6$, say $5\frac{1}{2}$ in.

The bond stress at r is

$$u = \frac{V}{jdO} = \frac{9{,}320}{.896 \times 22 \times \frac{12}{5.5} \times 2.75} = 79 \text{ lb. per sq. in.,}$$

which is below the permissible bond stress of 80 lb. per sq. in.

The rods in the heel cantilever will be carried beyond the back of the stem for a distance of $50 \times \frac{7}{8} = 44$ in. If 3 in. of concrete is allowed above the center of the steel, the total thickness of the base at the stem is 25 in.

As shown by the full line nw, Fig. 28 (a), the top of the heel projection is sloped. To determine the required thickness at the edge, the shear and bond stresses at intermediate points in the projection should be investigated.

The forces acting on the part of the heel cantilever to the right of a section 2 ft. from n are as follows: The weight of the masonry may be taken as $\frac{1{,}260}{5.58} \times 3.58 = 810$ lb.; the vertical component of the earth pressure as $\frac{8{,}190}{5.58} \times 3.58 = 5{,}250$ lb.; and the weight of the filling below the level sv as $\frac{7{,}590}{5.58} \times 3.58 = 4{,}870$ lb. The weight of the filling above the level sv is $3.58 \times \left(\frac{1.33 + 3.72}{2}\right) \times 100 = 900$ lb. Since the unit soil reaction at the section is $210 + \frac{3.58}{9} \times (4{,}590 - 210) = 1{,}950$ lb. per sq. ft., the total upward force is $3.58 \times \left(\frac{1{,}950 + 210}{2}\right) = 3{,}870$ lb. Hence, the shear at a section 2 ft. from n is

$$810 + 4{,}870 + 5{,}250 + 900 - 3{,}870 = 7{,}960 \text{ lb.}$$

At this section, the approximate required effective thickness of the heel projection is

$$d=\frac{V}{jbv}=\frac{7{,}960}{.896\times12\times40}=18.5 \text{ in.}$$

Then, $p=\frac{.601}{5.5\times18.5}=.0059$ and $j=.886$. Hence, a more correct value of the thickness is

$$d=\frac{7{,}960}{.886\times12\times40}=18.7 \text{ in.}$$

The unit bond stress is

$$u=\frac{V}{jdO}=\frac{7{,}960}{.886\times18.7\times\frac{12}{5.5}\times2.75}=80 \text{ lb. per sq. in.}$$

which is satisfactory. If the heel projection is sloped so that the effective thickness is 22 in. at the back of the stem and 18.7 in. at a point 2 ft. from n, the required effective thickness at the edge of the heel cantilever is

$$22-\frac{5.58}{2}\times(22-18.7)=12.8, \text{ say } 13 \text{ in.}$$

and the total thickness is 16 in.

The required steel area in the toe cantilever per in. of wall is, approximately,

$$A_s=\frac{M'}{f_sjd}=\frac{8{,}080}{16{,}000\times.857\times22}=.0268 \text{ sq. in.}$$

Since the corresponding value of p is $\frac{.0268}{22}=.00122$, for which $j=.942$, a more accurate value of the steel area is

$$A_s=\frac{8{,}080}{16{,}000\times.942\times22}=.0244 \text{ sq. in. per in.}$$

If ½-in. square bars are used, the spacing required to resist the tensile stress is $.25\div.0244=10$ in., and the unit bond stress is

$$u=\frac{7{,}760}{.942\times22\times\frac{12}{10}\times2}=156 \text{ lb. per sq. in.}$$

In order to reduce this to 100 lb., which is the allowable value for deformed bars with ordinary anchorage, the spacing must be decreased to

$\frac{100}{156} \times 10 = 6.4$, say $6\frac{1}{4}$ in. These rods will be carried beyond the face of the stem for a distance of 50 times their side dimension or $50 \times \frac{1}{2} = 25$ in.

The longitudinal reinforcement in the base will consist of $\frac{1}{2}$-in. round rods placed as shown in Fig. 29 (*a*).

EXAMPLES FOR PRACTICE

1. A cantilever retaining wall having a total height of 18 feet is to retain material that weighs 110 pounds per cubic foot and has a slope of repose of $1\frac{1}{2}$ horizontal to 1 vertical. The surface of the filling is to slope upwards from the back of the wall at the slope of repose. If the face of the stem is to be located 2 feet 6 inches from the toe, and the stem is to be treated as a rectangle 20 inches wide and the base as a rectangle 24 inches deep, what approximate width of the base is required to resist overturning? Ans. 10 ft. 6 in.

2. For the wall in example 1, find (*a*) the required thickness of the concrete at the bottom of the stem to the next larger inch, assuming the maximum thickness of the base as 36 inches and allowing 2 inches of concrete for protecting the steel in the stem, and (*b*) the spacing of 1-inch plain round rods at that depth. Take $f_s = 16{,}000$, $f_c = 650$, and $f'_c = 2{,}000$ pounds per square inch and $n = 15$. Ans. $\begin{cases} (a) \ 23 \text{ in.} \\ (b) \ 5\frac{1}{4} \text{ in.} \end{cases}$

3. In the wall in examples 1 and 2, the top width of the stem is 12 inches, and the face is vertical. Determine (*a*) the maximum required thickness of the heel cantilever, allowing 3 inches of concrete above the steel, and (*b*) the spacing of $\frac{7}{8}$-inch plain round rods. Ans. $\begin{cases} (a) \ 39 \text{ in.} \\ (b) \ 5\frac{1}{4} \text{ in.} \end{cases}$

4. In the wall of the preceding examples, the effective thickness of the toe projection at the face of the stem is 36 inches and the average total thickness of the toe cantilever is 30 inches. Determine the required spacing of $\frac{1}{2}$-inch square deformed bars in the toe cantilever. Ans. $6\frac{1}{2}$ in.

DESIGN OF COUNTERFORT RETAINING WALL

66. Comparison of Cantilever With Counterfort Wall.—In a counterfort retaining wall the stem and the heel projection are tied together at intervals by means of counterforts so as to enable them to act as slabs supported by the counterforts. For this reason, less concrete and reinforcement are needed than in a cantilever wall of the same height. On the other hand, a cantilever wall requires less expensive form work and

can be more easily constructed than a counterfort wall. As a rough guide in choosing between the two types, it may be assumed that in localities where skilled labor is available, a counterfort wall is more economical than a cantilever wall for heights exceeding about 20 feet.

67. Method of Designing Stem.—In a counterfort wall, as in a cantilever wall, the earth pressure on the stem varies from a maximum at the bottom to zero at the top. Therefore, the greatest shear and bending moments occur at the bottom, and the maximum thickness of the stem and the maximum amount of reinforcement are required there. In designing the stem at the bottom, a strip 1 foot high is treated as a slab uniformly loaded with the pressure at the bottom of that strip and supported at the counterforts. This pressure is computed for a level fill without surcharge by applying formula 1, Art. 17, in which h_x is the depth to the bottom of the strip; for a sloping fill by formula 1, Art. 18; and for a level fill with a superimposed load by taking the sum of the values of p_x in formula 1, Art. 17, and p' in formula 1, Art. 19.

In comparatively low counterfort walls, where the required thickness at the bottom of the stem is not much greater than the practical minimum thickness at the top, the same thickness is usually kept for the entire height of the stem, the face and back being made vertical. For high walls, where the required thickness at the bottom is considerably greater than the practical minimum thickness, it is best to taper the stem in order to effect a saving of concrete. After the thickness of the stem has been selected, the spacing of the main reinforcement may be established to allow for the variation in bending moment. It is customary to change the spacing only at intervals and to maintain the same spacing for about the upper half of the exposed part of the stem.

When the thickness of the stem is uniform throughout its height, the required reinforcement in each 1-foot strip is proportional to the bending moment in that strip. Furthermore, if the fill is level and carries no superimposed load, or if its surface is inclined, the bending moment, and therefore the

steel area, varies as the distance from the top of the wall to the bottom of the strip in question. Since the spacing of the reinforcement is inversely proportional to the required area of steel, the height at which a specified spacing is allowable may be computed by the formula

$$h_y = h_s \times \frac{s}{y}$$

in which h_y = height, in feet, from top of stem to point at which a certain spacing is to be used;

h_s = total height of stem, in feet;

s = spacing at bottom of stem, in inches;

y = desired spacing of main reinforcement at height h_y, in inches.

For example, if the height of the stem is 25 feet and the required spacing of the reinforcing rods at the bottom of the stem is 5 inches, the height below the top at which the spacing may be increased to 6 inches is $h_y = 25 \times \frac{5}{6} = 20.8$ feet.

68. In a counterfort retaining wall the stem slab may be assumed to be continuous over the counterforts. Some engineers, however, design the stem as a slab simply supported by the counterforts. When continuity is assumed, the value of the maximum positive or negative bending moment is taken as $\frac{Wl}{12}$ in each intermediate panel and $\frac{Wl}{10}$ in each end panel, W being the total load on a strip of slab 1 foot high and l the span or clear distance between counterforts. The positive reinforcement in the middle of the span is placed near the face of the stem, as in Fig. 20 (a). Negative reinforcement at the counterforts is provided either by inserting separate rods near the back of the stem, as in (b), or by bending every alternate rod of the positive reinforcement at the point of inflection and extending it to or beyond the nearer point of inflection in the adjacent span, as in (c). In either method, the rods for negative reinforcement should project far enough beyond each face of the counterfort to develop their tensile strength by bond.

The inflection points are commonly assumed to be located at a distance of two-tenths of the clear span from the faces of the counterforts. The critical sections for bond stress in the negative reinforcement are at the faces of the counterforts and those in the positive reinforcement are at the points of inflection. The maximum unit bond stress in the positive reinforcement of the stem slab is generally below the allowable value for ordinary anchorage. If the negative reinforcement is provided with special anchorage, the same size and spacing of reinforcing rods may usually be employed as for the positive reinforcement.

Since the stem is also fixed at its junction with the base and is not free to deflect horizontally, vertical rods should be provided near the back of the stem, as rods *d* in Fig. 20 (*a*), in order to prevent cracks between the stem and the base; the spacing of these rods should not exceed 18 inches, and their area should be not less than .3 per cent. of the effective area of the stem. All the secondary reinforcement *e* in the stem should be placed near the face; usually, $\frac{1}{2}$-inch round rods spaced about 24 inches on centers are sufficient to transmit the stresses in the slab to the main rods and to help prevent shrinkage and temperature cracks.

69. Design of Base.—The part of the base that constitutes the heel projection is designed either as a continuous slab or as a series of simple slabs supported by the counterforts. If the surface of the fill in back of the wall is level, the forces acting on the heel slab are the downward weights of the masonry and the fill and any superimposed load on it, and the upward soil reaction, which decreases from the back of the stem to the heel of the wall. When the fill is surcharged, each strip supports also a downward load due to the vertical component of the part of the earth pressure that acts on the base, as explained for cantilever walls.

Since the soil reaction, which is subtracted from the downward loads, is least at the heel of the wall, the shear and bending moment in the heel projection are greatest at the edge of the base. Hence, the thickness of the base is determined by

considering a strip 1 foot wide at the heel of the wall. This thickness is kept constant for the entire heel projection, but the area of reinforcement may be reduced toward the stem. The shear and bending moment at any section of the projection are proportional to the resultant load per square foot at the section, and the allowable spacing of the reinforcement at any point is equal to the spacing at the heel multiplied by the ratio of the load per square foot at the heel to the load per square foot at the point in question. The positive reinforcement in the heel slab is placed near the bottom and the negative reinforcement, which may be provided in the same manner as in the stem, is near the top at the counterforts.

The toe projection is usually designed as a cantilever in the same manner as for a cantilever wall. However, for high walls, an appreciable saving in materials may sometimes be effected by continuing the lower part of each counterfort beyond the face of the wall to form a buttress and by designing the toe projection as a slab supported by these buttresses.

70. Design of Counterforts. — The counterforts are designed as vertical cantilevers supported by the base and loaded with the earth pressure transferred by the stem. They are generally assumed to be of **T** section, part of the stem slab constituting the flange. The main reinforcement of each counterfort consists of inclined rods, f in Fig. 20 (a), which are placed near the back of the counterfort to resist the tensile stresses.

When the reinforcement is inclined to the vertical, the required steel area may be determined by the formula

$$A_s = \frac{M}{f_s jd \cos Y} \qquad (1)$$

in which A_s = required steel area, in square inches;

M = bending moment at any horizontal section, in inch-pounds;

f_s = tensile unit stress, in pounds per square inch;

jd = distance, in inches, from center of stem to center of steel area, measured along horizontal section;

Y = angle of inclination of reinforcement to vertical.

The unit bond stress in the reinforcement can be found by the formula

$$u = \frac{V}{jdO \cos Y} \qquad (2)$$

in which u = unit bond stress, in pounds per square inch;
V = shear at section, in pounds;
O = sum of perimeters of reinforcing bars, in inches.

The load on a counterfort varies from a maximum at the base to zero at the top, and therefore some of the main rods may be stopped at points where they are no longer needed. When the fill is level and has no superimposed load, or its surface slopes, the bending moment at any horizontal section through the counterfort is proportional to the cube of the depth of the section below the top, as in the case of the stem of a cantilever wall. Since the lever arm of the stress couple varies directly with this depth, the required area of steel is proportional to the square of the depth. Hence, the theoretical height at which the steel area may be reduced to a certain fraction of the maximum value can be found by the formula

$$h_x = h_c \sqrt{\frac{A_x}{A_s}} \qquad (3)$$

in which h_x = height, in feet, from top of counterfort to horizontal section at which a certain steel area is to be provided;
h_c = height of counterfort, in feet;
A_x = steel area, in square inches, to be provided at height h_x;
A_s = required steel area at bottom of counterfort, in square inches.

The theoretical ends of the rods are located by measuring down the computed distance h_x vertically. The rods should then be carried above the theoretical point of stoppage far enough to develop by bond about half of their tensile strength.

71. Horizontal stirrups, g in Fig. 20 (a), are needed in order to tie the stem to the counterforts and to provide web reinforce-

ment in them; therefore, they should be strong enough to resist the entire reaction of the stem slab. These stirrups should be looped around the inclined tensile reinforcement of the counterfort, and hooked at their ends to either the horizontal rods near the face of the stem, as in (*a*), or to distributing rods, as in (*c*). The counterforts should also be provided with vertical stirrups *h* to tie the base to them. These stirrups should be sufficiently strong to carry the resultant downward load on the base; they should be looped around the inclined rods of the counterfort and hooked at their ends to the rods of the slab.

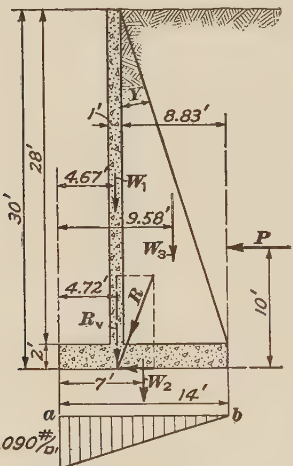


FIG. 30

The thickness of each counterfort should be sufficient to resist the shearing stresses and to offer ample room for the reinforcement in the counterfort. Generally, a thickness of not less than 12 inches is found desirable. The spacing of counterforts in the usual designs ranges from 8 to 12 feet. Closer spacing of the counterforts reduces the bending moments in the stem and heel slabs and effects a saving in materials, but increases the cost of form work. The most desirable spacing, of course, is the one for which the cost of the wall is lowest.

72. Construction of Counterfort Walls.—In constructing a counterfort wall, the stem slab and counterforts should always be poured together in one continuous operation. When necessary, a construction joint may be provided at the junction of the base to the stem and counterforts. The vertical expansion joints in the stem should be placed not over 60 feet apart. Usually, it is found more economical to locate the expansion joints midway between counterforts. The span in which such a joint occurs is then treated as two cantilevers supported by the counterforts. Sometimes, the expansion

joints are located at counterforts, and the adjacent spans of the stem slab are designed as the end spans of continuous beams.

73. Illustrative Design.—The design of a counterfort retaining wall is illustrated in the following example.

EXAMPLE.—A reinforced-concrete retaining wall of the counterfort type is to have a total height of 30 feet and is to support a level bank of earth whose weight is 110 pounds per cubic foot and whose slope of repose is 1½ horizontal to 1 vertical. The center of the stem is to be located at the edge of the middle third of the base and the counterforts are to be spaced 10 feet on centers. The allowable pressure on the soil is 4 tons per square foot and the coefficient of friction of the wall on the foundation is .6. Design an intermediate panel of the wall, assuming $f_s=18{,}000$, $f_c=800$, and $f_c'=2{,}000$ pounds per square inch, and $n=15$.

SOLUTION.—*Width of Base:* As the dimensions of the different parts of the wall are unknown, tentative dimensions are first assumed in order to determine the proper width of base. As shown in Fig. 30, the thickness of the stem will be taken as 1 ft., that of the base slab as 2 ft., and the width of the base as $.45\times30=13.5$, say 14 ft.

For a slope of repose of 1½:1, the earth pressure is, by formula 4, Art. 17,

$$P=.143wh^2=.143\times110\times30^2=14{,}160 \text{ lb.}$$

Since the bank is level, the pressure acts horizontally at a distance of $\frac{1}{3}\times30=10$ ft. above the bottom of the base, and the overturning moment is $14{,}160\times10=141{,}600$ ft.-lb.

For the assumed dimensions, the weights, lever arms, and moments of the concrete and filling for a section of wall of 1 ft. length are as tabulated below:

PART	WEIGHT, IN LB.	ARM, IN FT.	MOMENT, IN FT.-LB.
Stem	$W_1=1\times28\times150 \quad = 4{,}200$	$\frac{1}{3}\times14=4.67$	19,600
Base	$W_2=14\times2\times150 \quad = 4{,}200$	$\frac{1}{2}\times14=7$	29,400
Filling	$W_3=8.83\times28\times110=27{,}200$	$14-\frac{1}{2}\times8.83=9.58$	260,600
Total	35,600		309,600

The moment of the resultant R is $309{,}600-141{,}600=168{,}000$ ft.-lb. The vertical component R_v of the resultant is 35,600 lb., and the distance from the toe to the point where the resultant cuts the base is $\dfrac{168{,}000}{35{,}600}=4.72$ ft., which is only about ½ in. greater than $\frac{1}{3}\times14=4.67$ ft. Hence, the assumed width of base is satisfactory.

Sliding: The force tending to cause the wall to slide is the earth pressure, or 14,160 lb., and the frictional resistance is $35{,}600\times.6=21{,}360$ lb. Since the factor of safety is $\frac{21{,}400}{14{,}160}=1.51$, the wall may be considered stable without additional provision against sliding.

Pressure on Foundation: The resultant cuts the base so close to the edge of the middle third that the soil reaction at the heel may be taken as zero and that at the toe as $\frac{2R_v}{d}=\frac{2\times 35{,}600}{14}=5{,}090$ lb. per sq. ft. The distribution of the soil pressure is then represented by the triangle *abc*. Since the allowable pressure on the foundation is 4 tons per sq. ft., the assumed width of base will be adopted.

Stem Slab: For the purpose of designing the stem, the thickness of the base is assumed to be 2 ft. and that of the counterforts 12 in. The earth pressure per sq. ft. on the bottom strip of stem 1 ft. high is that corresponding to a height of 28 ft., which by formula 1, Art 17, is

$$p_x=wh_x\frac{1-\sin Z}{1+\sin Z}=110\times 28\times\frac{1-.555}{1+.555}=881\text{ lb.}$$

Since the distance between centers of counterforts is 10 ft., the clear span of the stem slab is $10-1=9$ ft. The total load W on the bottom strip is $881\times 9=7{,}930$ lb., the maximum shear is $\frac{W}{2}=3{,}970$ lb., and the maximum positive or negative bending moment in an intermediate panel is $\frac{Wl}{12}=\frac{7{,}930\times 9}{12}=5{,}950$ ft.-lb. For $f_c=800$, $f_s=18{,}000$, and $n=15$, the value of the constant K is 138.7, and the approximate value of j is .867. Hence, the approximate effective thickness of the stem required to resist the bending moment is

$$d=\sqrt{\frac{M'}{K}}=\sqrt{\frac{5{,}950}{138.7}}=6.55\text{ in.}$$

If the reinforcement is provided with special anchorage, the allowable shearing unit stress is $.03f'_c=.03\times 2{,}000=60$ lb. per sq. in., and the effective thickness required to resist the shear at the bottom of the stem is

$$d=\frac{V}{jbv}=\frac{3{,}970}{.867\times 12\times 60}=6.35\text{ in.}$$

The minimum practical total thickness of 12 in. is here adopted for the entire height of stem, as in Fig. 31 (*a*), and if 2 in. is allowed for protection of the main reinforcement, the corresponding effective thickness is 10 in. The negative reinforcement will here be provided by

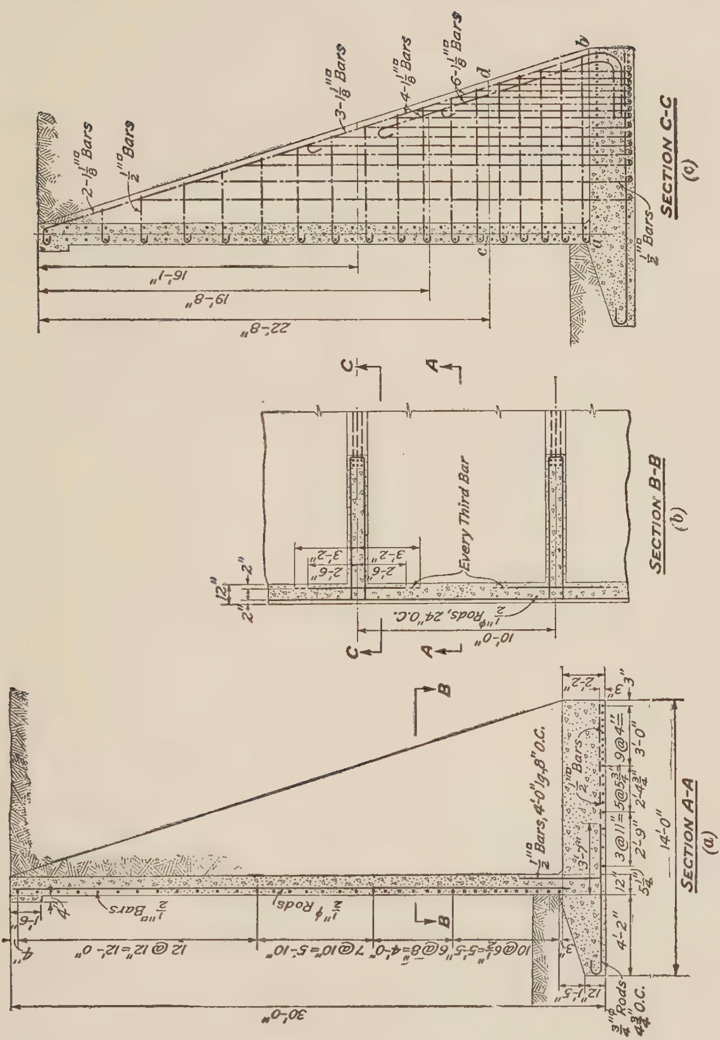

SECTION C-C
(c)
2-1⅛" Bars
½" Bars
3-1⅛" Bars
4-1⅛" Bars
6-1⅛" Bars
½" Bars
16'-1"
19'-8"
22'-8"
SECTION B-B
(b)
Every Third Bar
½" φ Rods, 24" O.C.
10'-0"
3'-2"
2'-6"
12"
2"
SECTION A-A
(a)
½" Bars, 4'-0" lg, 8" O.C.
½" Bars
½" φ Rods
½" Bars
¾" φ Rods 4¾" O.C.
12 @ 12" = 12'-0"
7 @ 10" = 5'-10"
6 @ 8" = 4'-0"
10 @ 6½" = 5'-5"
30'-0"
14'-0"
4'-2"
3'-0"
3 @ 11" = 2'-9"
5 @ 5¾" = 2'-4¾"
9 @ 4"
2'-2"
3'-7"
1'-5"
1'-6"

FIG. 31

means of separate rods, as shown in (b). At the bottom of the stem, the approximate steel area required per in. of height is

$$A_s = \frac{M'}{f_s jd} = \frac{5{,}950}{18{,}000 \times .867 \times 10} = .0381 \text{ sq. in.}$$

Then, $p = \frac{.0381}{10} = .00381$, $j = .905$, and the more exact steel area is

$$A_s = \frac{5{,}950}{18{,}000 \times .905 \times 10} = .0365 \text{ sq. in.}$$

If $\frac{1}{2}$-inch square bars are used, the cross-sectional area of each bar is .25 sq. in., and the required spacing is $\frac{.25}{.0365} = 6.85$, say $6\frac{3}{4}$ in.

In the positive reinforcement, the critical sections for bond stress are at the inflection points, which are located at a distance of $.2 \times 9 = 1.8$ ft. from the faces of the counterforts, or $4.5 - 1.8 = 2.7$ ft. from the center of span. The shear at those points is $881 \times 2.7 = 2{,}380$ lb., and the maximum unit bond stress is

$$u = \frac{V}{jdO} = \frac{2{,}380}{.905 \times 10 \times \frac{12}{6.75} \times 2} = \frac{2{,}380}{32.2} = 74 \text{ lb. per sq. in.}$$

which is satisfactory for plain bars with ordinary anchorage. In the negative reinforcement, the critical sections for bond stress are at the faces of the counterforts where the shear is 3,970 lb. The unit bond stress is $\frac{3{,}970}{32.2} = 123.3$ lb. per sq. in. In order to reduce this stress to 120 lb. per sq. in., which is the allowable value for plain bars provided with special anchorage, the spacing should be changed to $\frac{120}{123.3} \times 6.75 = 6.57$, say $6\frac{1}{2}$ in. Two-thirds of the negative reinforcement is carried to the points of inflection, or $.5 + 1.8 = 2.3$ ft., say 2 ft. 6 in., on each side of the center of the counterfort. The other one-third is continued beyond the point of inflection far enough to develop by bond one-third of its tensile strength, or a distance of $\frac{1}{3} \times \frac{.25 \times 18{,}000}{2 \times 80} = 9.4$ in., say .8 ft. Hence, every third bar of the negative reinforcement extends $2.3 + .8 = 3.1$ ft., say 3 ft. 2 in. from the center of the counterfort.

By the formula of Art. 67, the height below the top of the stem at which the spacing of the rods may be increased to 8 in. is

$$h_y = h_s \times \frac{s}{y} = 28 \times \frac{6.5}{8} = 22.75 \text{ ft.}$$

A spacing of 10 in. is permissible at $28 \times \frac{6.5}{10} = 18.2$ ft.; and 12 in. at $28 \times \frac{6.5}{12} = 15.2$ ft. Hence, the spacing shown in Fig. 31 (*a*) is satisfactory.

The secondary reinforcement will consist of ½-in. round rods spaced about 24 in. on centers between counterforts. The vertical rods tying the back of the stem to the base should have an area of $.003 \times 10 = .03$ sq. in. per in. of wall, and ½-in. square bars spaced 8 in. on centers are satisfactory. They should extend about 50 times their side dimension, say 24 in., into the base and into the stem.

Heel Slab: In this case the upward reaction of the soil at the heel is zero. Hence, the load per sq. ft. on the strip of slab 1 ft. wide at the heel is the sum of its own weight and the weight of the earth directly above it, or $2 \times 150 + 28 \times 110 = 300 + 3{,}080 = 3{,}380$ lb. Since the span is 9 ft., the total load on the strip is $3{,}380 \times 9 = 30{,}400$ lb., the maximum shear is 15,200 lb., and the maximum bending moment is $\frac{30{,}400 \times 9}{12} = 22{,}800$ ft.-lb.

The approximate effective thickness required to resist the bending moment is $\sqrt{\frac{22{,}800}{138.7}} = 12.8$ in. and that required to resist the shear is $\frac{15{,}200}{.867 \times 12 \times 60} = 24.3$ in. In this case, 23 in. may be assumed, because j will be considerably greater than .867. Then, the approximate area of steel per in. of width is $\frac{22{,}800}{18{,}000 \times .867 \times 23} = .0635$ sq. in. and $p = \frac{.0635}{23} = .00276$. Hence, $j = .917$, and the effective thickness may be $\frac{15{,}200}{.917 \times 12 \times 60} = 23$ in. If 3 in. of concrete is allowed below the steel, the total thickness is 26 in.

The more exact area of steel is $\frac{22{,}800}{18{,}000 \times .917 \times 23} = \frac{22{,}800}{380{,}000} = .06$ sq. in. per in., which can be supplied by means of ½-in. square bars spaced $\frac{.25}{.06} = 4.17$, say 4 in. on centers. The bond stress in the negative reinforcement is $\frac{15{,}200}{.917 \times 23 \times \frac{12}{4} \times 2} = 120$ lb. per sq. in., which is satisfactory for plain bars with special anchorage.

Two-thirds of the negative reinforcement will be carried 2 ft. 6 in., and the other one-third 3 ft. 2 in., beyond the center of the counterfort.

At 3 ft. from the heel of the wall, the upward soil reaction is $\frac{3}{14}\times 5{,}090=1{,}090$ lb. per sq. ft. Therefore, the resultant load on a 1-ft. strip of the base at that point is $3{,}380-1{,}090=2{,}290$ lb. per sq. ft., and the allowable spacing of the reinforcing rods is $4\times\frac{3{,}380}{2{,}290}=5.9$, say $5\frac{3}{4}$ in.

At 6 ft. from the heel, the upward soil pressure is 2,180 lb. per sq. ft., the resultant load at that point is $3{,}380-2{,}180=1{,}200$ lb. per sq. ft., and the allowable spacing is $4\times\frac{3{,}380}{1{,}200}=11.27$, say 11 in. Hence, the spacing shown in Fig. 31 (*a*) is satisfactory.

Toe Cantilever: The upward soil reaction at the face of the stem is $\frac{9.83}{14}\times 5{,}090=3{,}570$ lb. per sq. ft., and the total upward pressure per ft. of wall on the toe cantilever is $\frac{4.17}{2}\times(5{,}090+3{,}570)=18{,}060$ lb. If the cantilever is assumed to have an average thickness of 2 ft., its weight is $4.17\times 2\times 150=1{,}250$ lb. Hence, the maximum shear is $18{,}060-1{,}250=16{,}810$ lb.

The moment of the upward soil reaction about the face of the stem is $4.17\times 3{,}570\times\frac{4.17}{2}+\frac{1}{2}\times 4.17\times(5{,}090-3{,}570)\times\frac{2}{3}\times 4.17=39{,}900$ ft.-lb., the moment of the downward weight of the cantilever is $1{,}250\times\frac{4.17}{2}=2{,}600$ ft.-lb., and the maximum bending moment is $39{,}900-2{,}600=37{,}300$ ft.-lb.

Therefore, the approximate effective thickness required to resist the bending moment is $\sqrt{\frac{37{,}300}{138.7}}=16.4$ in., and that required to resist the shear is $\frac{16{,}810}{.867\times 12\times 60}=26.9$ in. For a trial thickness of 26 in., the approximate steel area is $\frac{37{,}300}{18{,}000\times .867\times 26}=.0918$ sq. in. per in., for which $p=\frac{.0918}{26}=.00353$, and $j=.908$. For the corrected value of j, the effective depth is $\frac{16{,}810}{.908\times 12\times 60}=25.7$, say 26 in., and a total depth of 29 in. is adopted. The required steel area now is $\frac{37{,}300}{18{,}000\times .908\times 26}=.0878$ sq. in., which can be provided by $\frac{3}{4}$ in. round bars spaced $\frac{.442}{.0878}=5.03$, say 5 in.

on centers. The unit bond stress is $\dfrac{16{,}810}{.908\times 26\times\frac{12}{5}\times 2.36}=126$ lb. per sq. in.

For a stress of 120 lb. per sq. in., which is satisfactory for plain bars with special anchorage, the spacing should be $\dfrac{120}{126}\times 5=4.76$, say $4\frac{3}{4}$ in. Each rod is hooked at the toe of the wall to provide special anchorage; also, in order that the rods may develop by bond their allowable tensile strength, they are extended beyond the face of the stem for a distance equal to $\dfrac{.442\times 18{,}000}{2.36\times 80}=42.1$ in., say 3 ft. 7 in. Every fourth rod is continued to the heel of the wall to provide secondary reinforcement in the heel slab.

Counterforts: By formula 4, Art. 17, the total earth pressure per foot of wall for a height of 27 ft. 10 in. is $.143\times 110\times 27.83^2=12{,}180$ lb. Since the counterforts are spaced 10 ft. on centers, the total pressure on each counterfort is $12{,}180\times 10=121{,}800$ lb. The point of application of that pressure is at a distance of $\frac{1}{3}\times 27.83=9.28$ ft. above the top of the base, and the maximum bending moment in the counterfort is $121{,}800\times 9.28=1{,}130{,}000$ ft.-lb. or 13,560,000 in.-lb.

The counterforts are designed as **T** beams, and the first step is to determine the width of web required to resist the shear. Since web reinforcement is to be used, the allowable shearing unit stress is $.06\times 2{,}000=120$ lb. per sq. in. Furthermore, if 4 in. of concrete from the center of the steel area to the back of the counterfort is allowed as protection, the maximum effective depth for shear is $14'\,0''-4'\,2''-4''=9$ ft. 6 in., or 114 in., and the approximate width b' required to resist the maximum shear of 121,800 lb. is $\dfrac{121{,}800}{120\times .875\times 114}=10.2$ in. Hence, the minimum practical width of 12 in. is satisfactory.

The distance ab from the center of the stem to the back of the counterfort at the top of the footing is $14'\,0''-4'\,2''-6''=9$ ft. 4 in., and if protection of 4 in. is allowed from the center of the steel area to the back of the counterfort, $jd=9$ ft. $=108$ in. Also, according to Fig. 30, $\tan Y=\dfrac{8.83}{28}=.315$, $Y=17°\,29'$, and $\cos Y=.954$. Hence, by formula 1, Art. 70, the area of steel required at the bottom of the counterfort is

$$A_s=\frac{M}{f_s jd\cos Y}=\frac{13{,}560{,}000}{18{,}000\times 108\times .954}=7.31 \text{ sq. in.}$$

which can be provided by means of six $1\frac{1}{8}$-in. square bars. By formula 2, Art. 70, the approximate bond stress in these bars is

$$u=\frac{V}{jdO\cos Y}=\frac{121{,}800}{108\times 6\times 4.5\times .954}=43.8 \text{ lb. per sq. in.}$$

which is quite low. The bars will be placed in two layers, as in Fig. 31 (*b*) and (*c*).

In this case the theoretical points at which some of the rods may be stopped can be located by applying formula 3, Art. 70. Thus, where two bars are stopped, $\frac{A_x}{A_s}=\frac{6-2}{6}=.667$, and the distance to the theoretical point of stoppage is

$$h_x=h_c\sqrt{\frac{A_x}{A_s}}=27.83\times\sqrt{.667}=22.7 \text{ ft., say } 22 \text{ ft. } 8 \text{ in.}$$

The theoretical ends of the bars are located in (*c*) by measuring 22 ft. 8 in. from the top of the wall along the center line of the stem to *c* and drawing the horizontal line *cd*. The bars are then carried beyond the point *d* a sufficient distance to develop by bond about half of their tensile strength or a distance of $\frac{1}{2}\times\frac{1.266\times18{,}000}{4.5\times80}=31.7$ in., say 2 ft. 8 in. These bars are provided with hooks at the ends for additional anchorage.

Where three bars remain, $\frac{A_x}{A_s}=\frac{3}{6}$ and $h_x=27.83\times\sqrt{.5}=19.7$ ft., say 19 ft. 8 in. Two bars are sufficient at a theoretical depth of $27.83\times\sqrt{.333}=16.1$ ft. or 16 ft. 1 in. The actual points of stoppage are located as in (*c*), the last two bars being carried to the top of the wall.

The next step in the design is the provision of horizontal stirrups to resist the entire reaction of the stem. At the bottom of the counterfort the intensity of pressure is, by formula 1, Art. 17,

$$p_x=wh_x\frac{1-\sin Z}{1+\sin Z}=110\times27.83\times\frac{1-.555}{1+.555}=876 \text{ lb. per sq. ft.}$$

and the reaction per ft. of height is $876\times9=7{,}880$ lb.

If the stirrups are made of $\frac{1}{2}$-in. square bars bent to **U** shape, as in (*b*), the available area is $2\times.25=.5$ sq. in. per stirrup and the required number of stirrups per ft. of height is $\frac{7{,}880}{.5\times18{,}000}=.88$. Hence, a spacing of 13 in. is adopted and the stirrups are hooked around alternate horizontal rods near the face of the stem, as in (*c*). Since the reaction decreases in proportion to the height, the spacing of the horizontal stirrups may be varied with that of the reinforcement in the stem. Thus, a stirrup is provided for every other rod all the way up the wall, as in (*c*).

The final step in the design is the provision of vertical stirrups for tying the heel slab to the counterfort. The 1-ft. strip of slab at the heel may be considered adequately tied by means of the main reinforce-

ment from the counterfort. For the second strip of slab the average upward soil reaction is $\frac{1.5}{14} \times 5{,}090 = 550$ lb. per sq. ft., the resultant downward load is $3{,}380 - 550 = 2{,}830$ lb. per sq. ft., and the load transferred to the counterfort is $2{,}830 \times 9 = 25{,}500$ lb. If the ties are spaced so that they can be hooked to the main reinforcement in the heel projection, the spacing in the second strip is 4 in. Then, the required area of each tie is $\frac{25{,}500}{18{,}000 \times \frac{12}{4}} = .472$ sq. in. Since the ties are bent to **U** shape around the main reinforcement of the counterfort, each leg should have an area of .236 sq. in., which can be provided by ½-in. square bars. The required area of a tie at any point is proportional to the resultant downward load on the strip of the heel projection directly below. Therefore, the ties may consist of ½-in. square bars spaced the same as the main slab reinforcement throughout the width of the counterfort, as in (*c*).

EXAMPLES FOR PRACTICE

1. A retaining wall with counterforts is to hold back a bank of earth level with its top. The total height of the wall is 36 feet, the weight of the filling is 115 pounds per cubic foot, and its slope of repose is 1½ horizontal to 1 vertical. The width of the base is to be 16 feet 9 inches, the center of the stem is to be located over the edge of the middle third of the base, and the counterforts are to be 10 feet on centers. If the stem is to be designed as a continuous slab, the thickness of the counterforts is assumed as 16 inches, and the thickness of the base as 3.5 feet, determine (*a*) the thickness of the stem in an intermediate span to the next larger inch, allowing 2 inches of concrete outside of the center of the steel; and (*b*) the spacing of ½-inch square deformed bars at the bottom of the stem, the negative reinforcement being provided by separate rods with ordinary anchorage. Assume $f_s = 16{,}000$, $f_c = 650$, and $f'_c = 2{,}000$ pounds per square inch, and $n = 15$.

Ans. { (*a*) 13 in.
(*b*) 5 in.

2. If the center of the stem in example 1 is placed 5 feet 7 inches from the toe of the wall, and the stem is 13 inches thick in its entire height, find (*a*) the vertical component of the resultant of all forces acting on the wall; (*b*) the distance from the toe to the point where the resultant cuts the base of the wall; (*c*) the maximum soil pressure at the toe; (*d*) the minimum soil pressure at the heel.

Ans. { (*a*) 53,800 lb.
(*b*) 5.61 ft.
(*c*) 6,390 lb. per sq. ft.
(*d*) 30 lb. per sq. ft.

3. For the wall in examples 1 and 2, find (*a*) the thickness of the heel projection to the next larger inch, allowing 3 inches of concrete below the center of the steel; (*b*) the spacing of ½-inch square deformed bars at the heel, negative reinforcement being provided by separate bars with ordinary anchorage.

$$\text{Ans.}\begin{cases}(a) & 44 \text{ in.}\\ (b) & 5 \text{ in.}\end{cases}$$

4. How many 1¼-inch square bars are needed to resist the bending moment at the bottom of a counterfort in the wall of the preceding examples? Ans. 8